

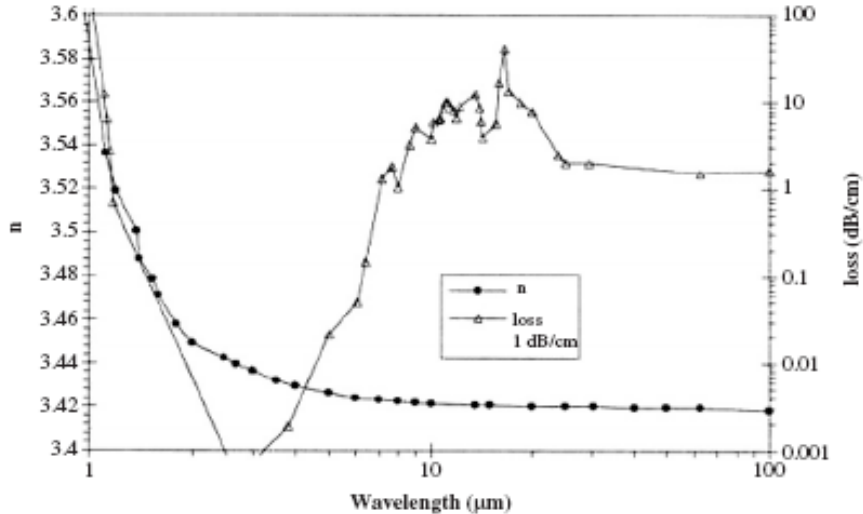


Lecture: *Silicon Photonics Waveguides*

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University of Surrey, Guildford, UK

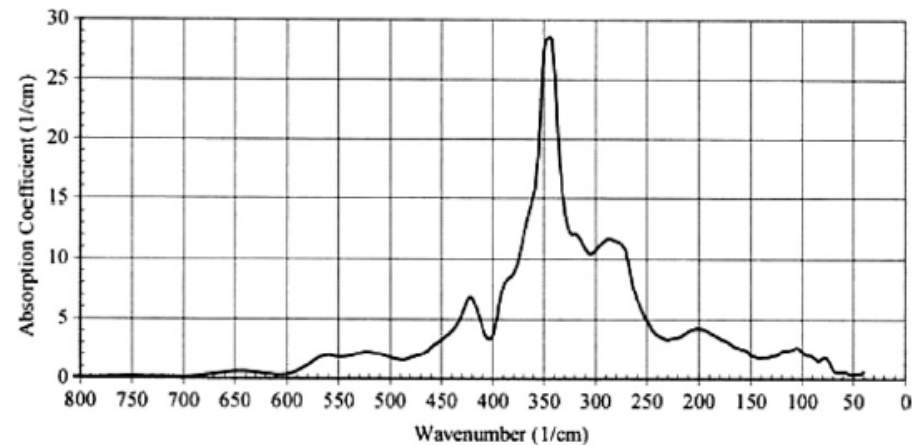
- Waveguide materials
- Planar waveguides
- Rib and ridge waveguides
- Strip waveguides
- Losses
- Polarisation issues
- Effect of stress

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Refractive index and absorption loss of Si
as a function of wavelength
(1.2 - 8 μm) + (24 - 200 μm)

Absorption coefficient of Ge as a function of wavenumber
(1.9 - 16.8 μm) + (140 - 200 μm)

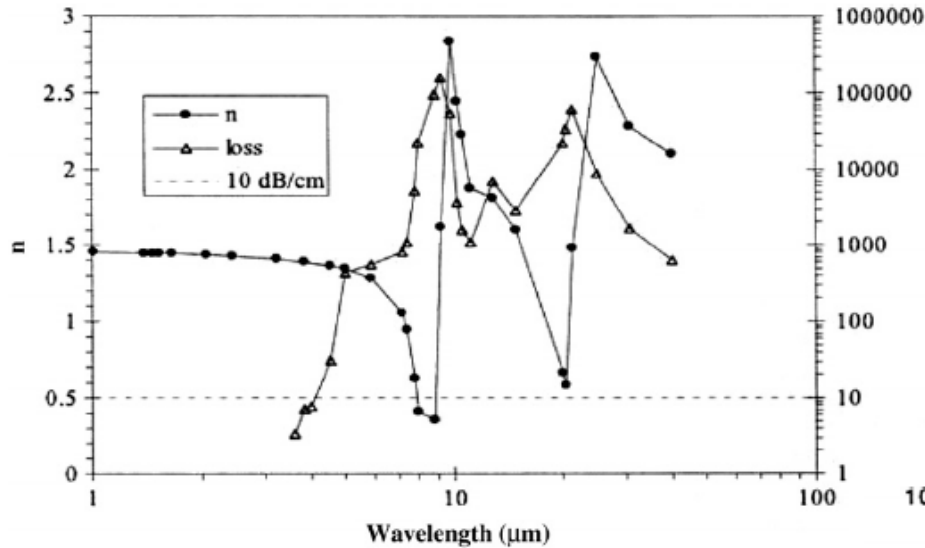


[Palik E D 1985 *Handbook of Optical Constants of Solids* vol 1
(New York: Academic)]

Si₃N₄: 1.2 - 6.7 μm

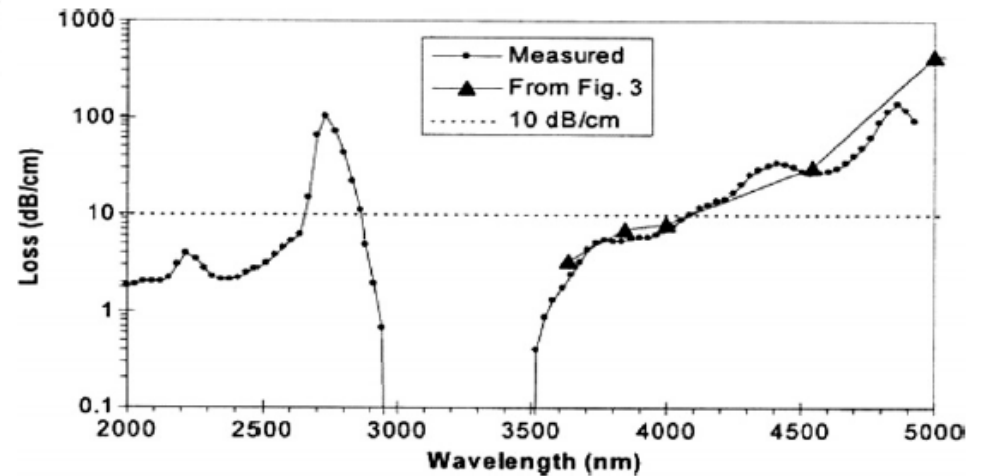
Al₂O₃: 1.2 - 4.4 μm

[Hawkins G J 1998 *Spectral characteristics of infrared optical materials and filters*
PhD Thesis Department of Cybernetics, University of Reading, UK]



Refractive index and absorption loss of SiO₂ as a function of wavelength

Absorption loss of SiO₂ as a function of wavelength (2-5 μm)

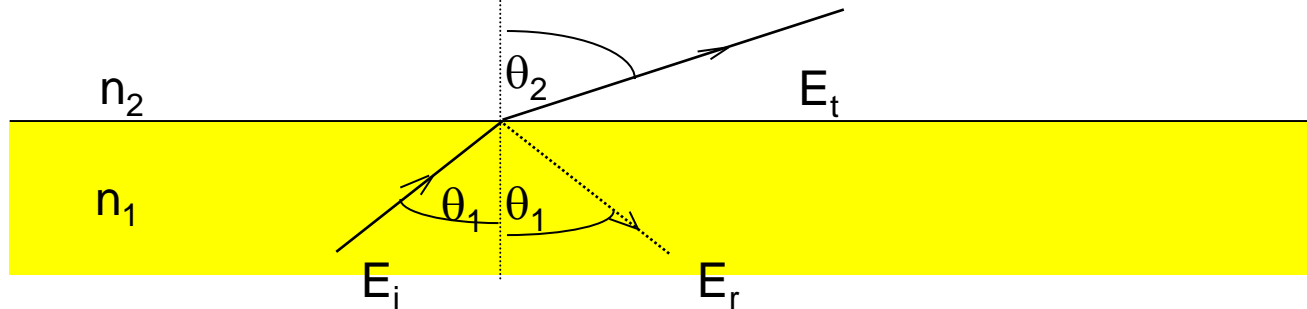


[Palik E D 1985 *Handbook of Optical Constants of Solids* vol 1 (New York: Academic)]

[R. Soref et al, J. Opt. A: Pure Appl. Opt, 8 (840) 2006]

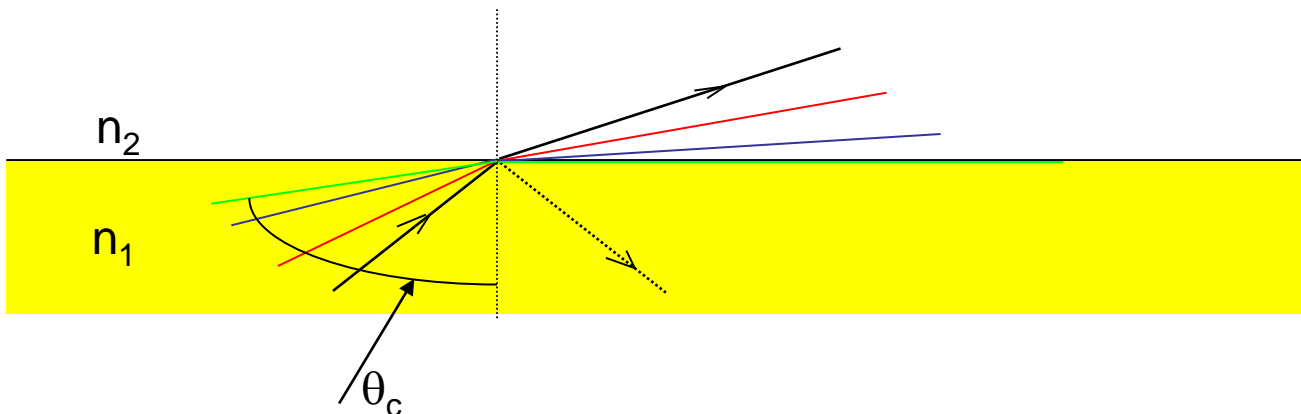
- Waveguide materials
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The Ray Optics Approach to Describing Planar Waveguides



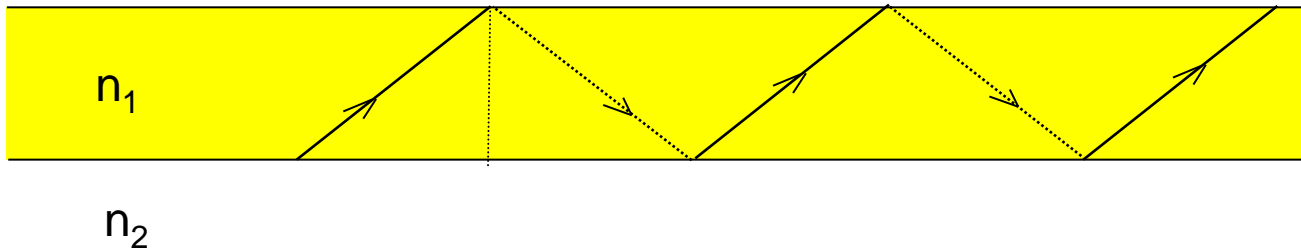
Light rays refracted and reflected at the interface of two media

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



$$n_1 \sin \theta_1 = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1}$$



Total internal reflection at two interfaces demonstrating the concept of a waveguide

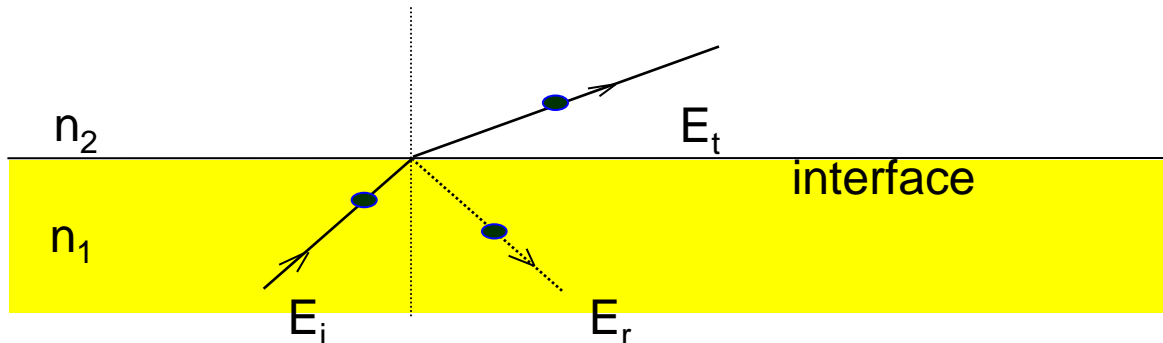
Reflection Coefficients

$$E_r = r.E_i$$

where 'r' is a complex reflection coefficient, which is polarisation dependant

The Transverse Electric (TE) condition is defined as the condition when the electric fields of the waves are perpendicular to the plane of incidence.

Correspondingly, the Transverse Magnetic (TM) condition occurs when the magnetic fields are perpendicular to the plane of incidence.



Circles indicate that the electric fields are vertical (i.e. coming out of screen)
Orientation of electric fields for TE incidence at the interface between 2 media.

The reflection coefficients r_{TE} and r_{TM} , are described by the Fresnel formulae

For TE polarisation:

$$r_{TE} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

For TM polarisation:

$$r_{TM} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

Using Snell's Law:

$$r_{TE} = \frac{n_1 \cos \theta_1 - \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}}{n_1 \cos \theta_1 + \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}}$$

And

$$r_{TM} = \frac{n_2^2 \cos \theta_1 - n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}}{n_2^2 \cos \theta_1 + n_1 \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1}}$$

If θ_1 is greater than θ_c

$$\mathbf{r} = \exp(\mathbf{j}\phi)$$

and

Where ϕ_{TE} and ϕ_{TM} are given by:

$$|\mathbf{r}| = 1$$

$$\phi_{\text{TE}} = 2 \tan^{-1} \frac{\sqrt{\sin^2 \theta_1 - \left(\frac{n_2}{n_1}\right)^2}}{\cos \theta_1}$$

and

$$\phi_{\text{TM}} = 2 \tan^{-1} \frac{\sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_1 - 1}}{\frac{n_2}{n_1} \cos \theta_1}$$

Strictly ϕ_{TE} and ϕ_{TM} should be $-ve$, but by convention these are usually taken as $+ve$
Silicon Photonics –PhD course prepared within FP7-224312 Helios project

But r relates reflected fields. Power is described by the Poynting vector

$$S = \frac{1}{Z} E^2 = \sqrt{\frac{\epsilon_m}{\mu_m}} E^2$$

And reflected power is related by reflectance R :

$$R = \frac{S_r}{S_i} = \frac{E_r^2}{E_i^2} = r^2$$

where E is electric field, ϵ_m is the permittivity of the medium, μ_m is the permeability of the medium, and Z is the impedance of the medium

Phase of a propagating wave and its wavevector

Let

$$E = E_0 \exp[j(kz \pm \omega t)]$$

and

$$H = H_0 \exp[j(kz \pm \omega t)]$$

Where z is the direction of propagation

Therefore, phase ϕ is: $\phi = kz \pm \omega t$

The phase varies with time (t), and with distance (z). These variations are quantified by taking the time derivative and the spatial derivatives:

$$\left| \frac{\partial \phi}{\partial t} \right| = \omega = 2\pi f \quad ; \quad \frac{\partial \phi}{\partial z} = k$$

where ω is angular frequency (rads/sec), and f is frequency (Hz).

\mathbf{k} is the wavevector (propagation constant) in the direction of the wavefront. It is related to wavelength, λ , by:

$$\mathbf{k} = \frac{2\pi}{\lambda}$$

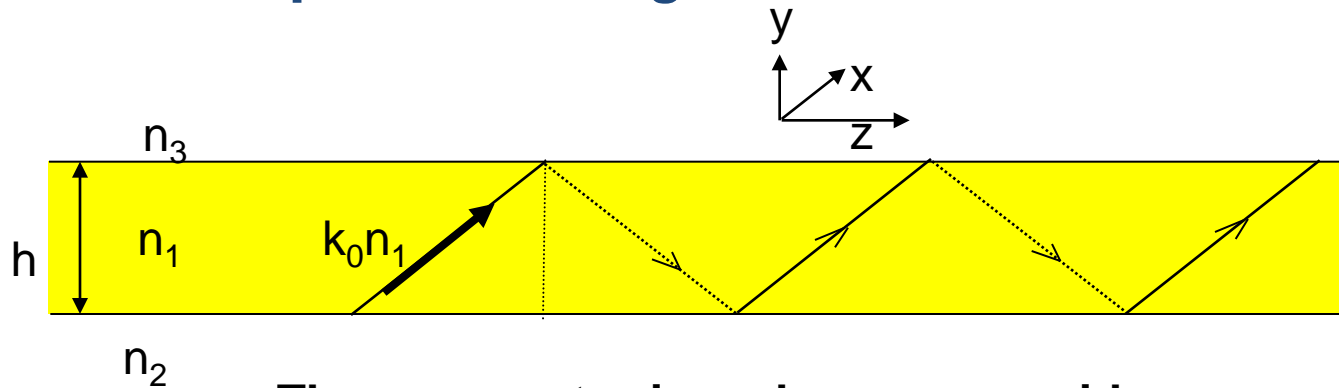
In free space $\mathbf{k} = \mathbf{k}_0$, and

$$k = nk_0$$

Hence in free space,

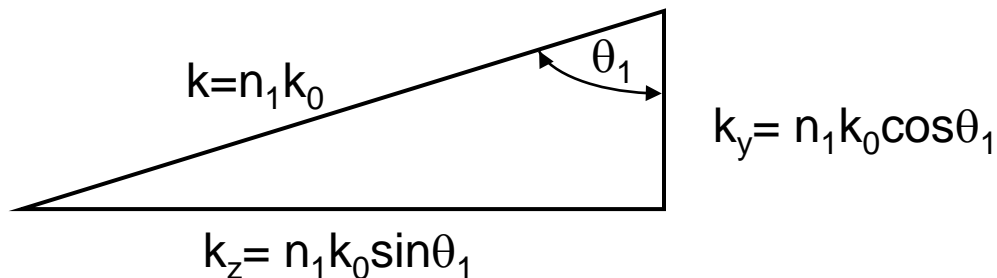
$$k_0 = \frac{2\pi}{\lambda_0}$$

Modes of a planar waveguide



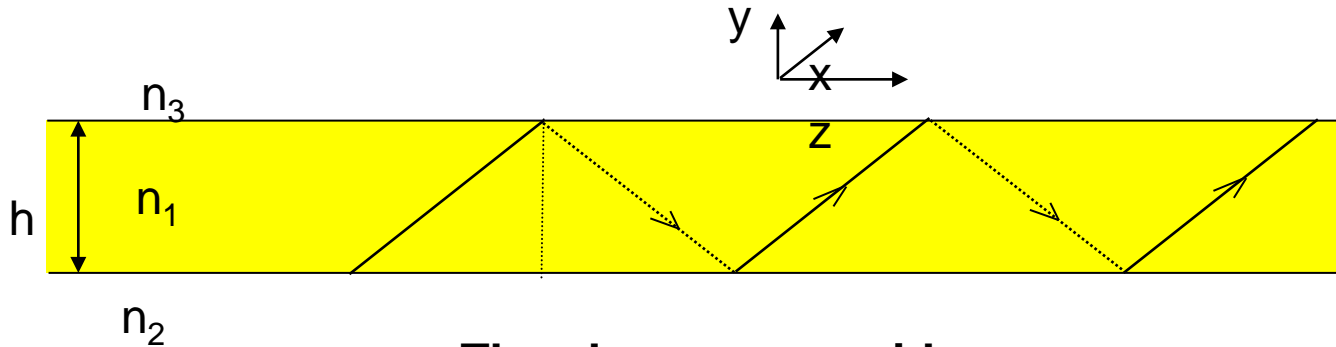
The wavevector in a planar waveguide

We can decompose the wavevector k , into two components, in the y and z directions.



The relationship between propagation constants in the y , z , and wavenormal directions

Consider propagation in the y direction



The planar waveguide

$$\phi_h = 2k_y h = 2k_0 n_1 h \cos \theta_1$$

We have seen that reflection coefficients give a phase change upon reflection. Let us refer to these phase shifts as ϕ_u , and ϕ_ℓ respectively. Hence the total phase shift is:

$$\phi_t = 2k_0 n_1 h \cos \theta_1 - \phi_u - \phi_\ell$$

For self this total phase shift must be a multiple of 2π ,

hence:

$$2k_0 n_1 h \cos \theta_1 - \phi_u - \phi_\ell = 2m\pi$$

Thus light propagates in discrete modes described by the polarisation and the mode number.

E.g. TE_0 , TE_1 , TM_0 , etc

Each mode will have a unique propagation constant in the y and z directions

The number of modes is limited by satisfaction of the requirements of total internal reflection.

The symmetrical planar waveguide

In the symmetrical planar waveguide, $n_2 = n_3$, and hence $\phi_u = \phi_\ell$. Therefore for TE polarisation, the equation becomes:

$$2k_0 n_1 h \cos \theta_1 - 4 \tan^{-1} \left[\frac{\sqrt{\sin^2 \theta_1 - \left(\frac{n_2}{n_1}\right)^2}}{\cos \theta_1} \right] = 2m\pi$$

This can be rearranged as :

$$\tan \left[\frac{k_0 n_1 h \cos \theta_1 - m\pi}{2} \right] = \left[\frac{\sqrt{\sin^2 \theta_1 - \left(\frac{n_2}{n_1}\right)^2}}{\cos \theta_1} \right]$$

Later we will solve this equation for angle θ_1

We can find the approximate number of modes supported by the waveguide as follows:

The minimum value that θ_1 can take is θ_c . i.e.

$$\sin \theta_c = \frac{n_2}{n_1}$$

Hence the right hand side of the previous equation reduces to zero and the equation becomes:

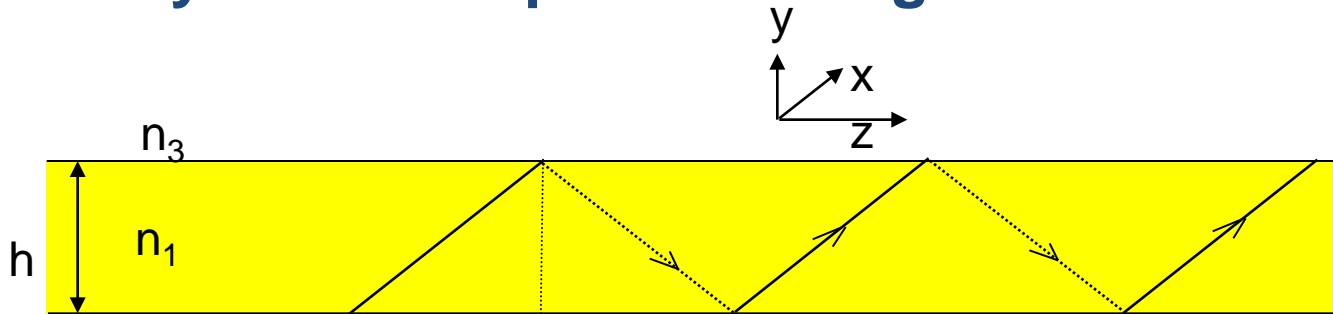
$$\frac{k_0 n_1 h \cos \theta_c - m_{\max} \pi}{2} = 0$$

rearranging for m , the mode number,

$$m_{\max} = \frac{k_0 n_1 h \cos \theta_c}{\pi}$$

Number of modes = $[m_{\max}]_{\text{int}} + 1$, since the lowest order mode (usually called the fundamental mode), has a mode number $m=0$. Note that the symmetrical waveguide is never cut-off.

The asymmetrical planar waveguide



n_2 Propagation in an asymmetric planar waveguide

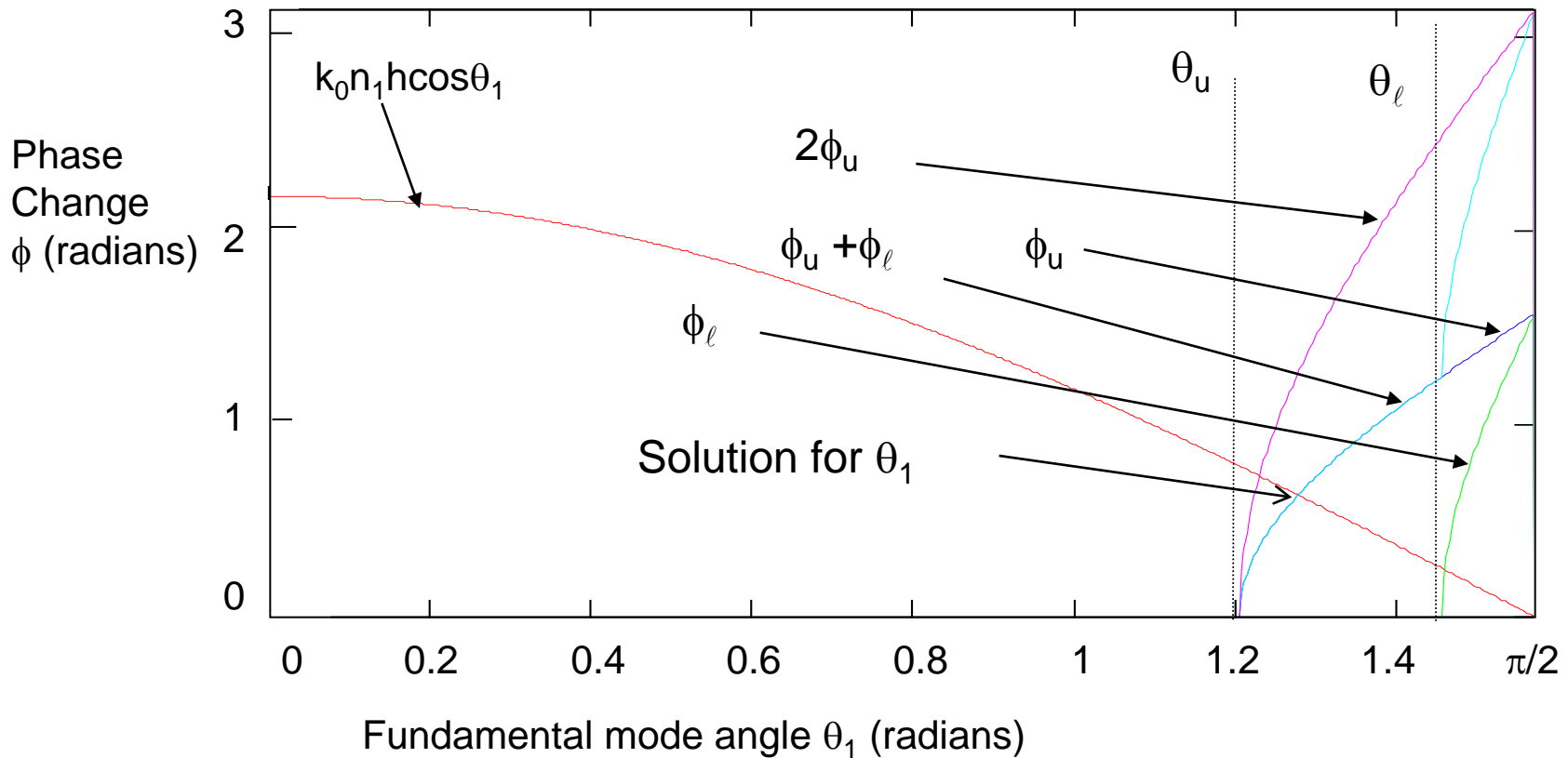
$n_2 \neq n_3$, and $\phi_u \neq \phi_l$, hence

$$[k_0 n_1 h \cos \theta_1 - m\pi] = \tan^{-1} \left[\frac{\sqrt{\sin^2 \theta_1 - \left(\frac{n_2}{n_1}\right)^2}}{\cos \theta_1} \right] + \tan^{-1} \left[\frac{\sqrt{\sin^2 \theta_1 - \left(\frac{n_3}{n_1}\right)^2}}{\cos \theta_1} \right]$$

Note that there is not always a solution for $m=0$, hence the asymmetrical guide may be cut-off.

Solving the eigenvalue equation for symmetrical and asymmetrical waveguides

Let $n_1 = 1.5$, $n_2 = 1.49$, $n_3 = 1.40$, $\lambda_0 = 1.3\mu\text{m}$, and $h = 0.3\mu\text{m}$, and TE polarisation

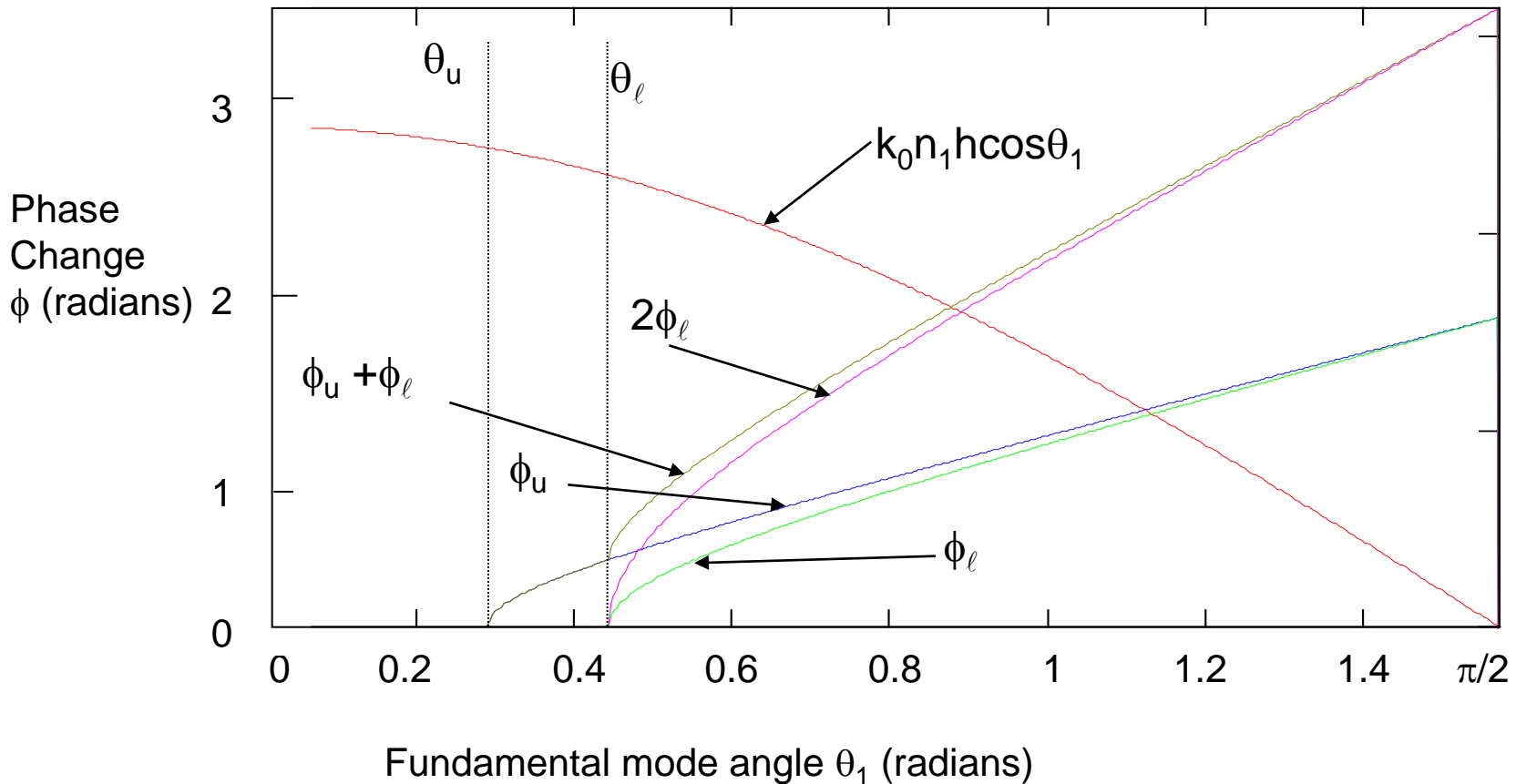


Solution of the eigenvalue equation for $m=0$

Note that the asymmetrical waveguide is cut-off, whereas the symmetrical waveguide is not

Now consider a silicon waveguide:

$n_1 = 3.5$ (silicon), $n_2 = 1.5$ (silicon dioxide), $n_3 = 1.0$ (air), $\lambda_0 = 1.3\mu\text{m}$, and $h = 0.17\mu\text{m}$.



Solution of the eigenvalue equation for $m=0$ (silicon-on-insulator)

Note that the symmetrical and asymmetrical waveguides are similar

Monomode Conditions

Consider the TE polarisation eigenvalue equation for a symmetrical waveguide:

$$\tan\left[\frac{k_0 n_1 h \cos \theta_1 - m\pi}{2}\right] = \left[\frac{\sqrt{\sin^2 \theta_1 - \left(\frac{n_2}{n_1}\right)^2}}{\cos \theta_1} \right]$$

For the second mode, $m=1$, and the equation becomes:

$$\tan\left[\frac{k_0 n_1 h \cos \theta_c - \pi}{2}\right] = 0$$

i.e.

$$\cos \theta_c = \frac{\pi}{k_0 n_1 h} = \frac{\lambda_0}{2n_1 h}$$

Hence for monomode conditions

$$\theta_c < \cos^{-1}\left(\frac{\lambda_0}{2n_1h}\right)$$

Effective Index

We have seen that propagation constants in the z and y directions are:

$$k_z = n_1 k_0 \sin \theta_1$$

and

$$k_y = n_1 k_0 \cos \theta_1$$

k_z is also known as β , the propagation constant in the direction of the waveguide

Now define a parameter N, called the effective index of the mode, such that:

$$N = n_1 \sin \theta_1$$

i.e.

$$k_z = \beta = Nk_0$$

The lower bound on β is determined by the larger of the critical angles of the waveguide, usually at the lower interface:

$$\beta \geq n_1 k_0 \sin \theta_\ell = k_0 n_2$$

The upper bound on β is governed by the maximum value of θ , which is 90° . In this case $\beta = k = n_1 k_0$. Hence:

$$k_0 n_1 \geq \beta \geq k_0 n_2$$

Substituting for $\beta = k = N k_0$:

$$n_1 \geq N \geq n_2$$

Just a little electromagnetic theory

Starting with Maxwell's equations, if we assume a loss-less, non conducting medium, limit ourselves to propagation in the z direction, and consider one polarisation at a time (TE or TM), we can derive a scalar equation describing wave propagation in our planar waveguide:

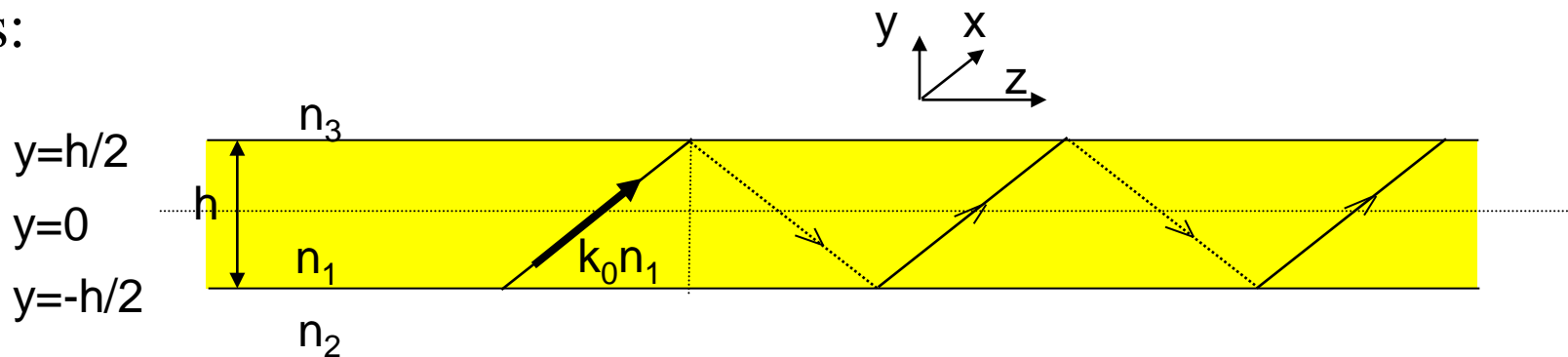
$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_m \epsilon_m \frac{\partial^2 E_x}{\partial t^2}$$

where ϵ_m is the permittivity of the waveguide, μ_m is the permeability of the waveguide, and in this case the electric field polarised in the x direction corresponds to TE polarisation. This is called the scalar wave equation.

Since there is only electric field in the x direction we can write the equation of this field as:

$$E_x = E_x(y)e^{-j\beta z}e^{j\omega t}$$

This means that there is a field directed (polarised) in the x direction, with a variation in the y direction yet to be determined, propagating in the z direction with propagation constant β , with sinusoidal ($e^{j\omega t}$) time dependence. The waveguide configuration is:



Propagation in an asymmetric planar waveguide

Solution of the wave equation provides us with expressions of the electric fields in the core and claddings:

In the upper cladding:

$$E_x(y) = E_u e^{-k_{yu}(y - \frac{h}{2})} \quad \text{For } y \geq (h/2)$$

In the core we will have

$$E_x(y) = E_c e^{-jk_{yc}y} \quad \text{For } -(h/2) \leq y \leq (h/2)$$

In the lower cladding we will have:

$$E_x(y) = E_\ell e^{k_{y\ell}(y + \frac{h}{2})} \quad \text{For } y \leq -(h/2)$$

Where $k_{yi}^2 = k_0^2 n_i^2 - \beta^2$

Note that n and k_y are written n_i and k_{yi} because they can now represent any of the three media (core, upper cladding, or lower cladding), by letting $i=1, 2, \text{ or } 3$.

Propagation constants again

In solving the wave equation we assumed field solutions with propagation constants in the core, and the upper and lower claddings, but we didn't discuss why the propagation constants took the form they did.

Our general solution was of the form:

$$\mathbf{E}_x = \mathbf{E}_c e^{-k_y y} e^{-j\beta z} e^{j\omega t}$$

However, in the three media, core, upper cladding and lower cladding, the propagation constant k_y took different forms. In the claddings k_y was a real number, whereas in the core k_y was an imaginary number

- total internal reflection occurs
- field decays in cladding
- part of the field propagates in the cladding

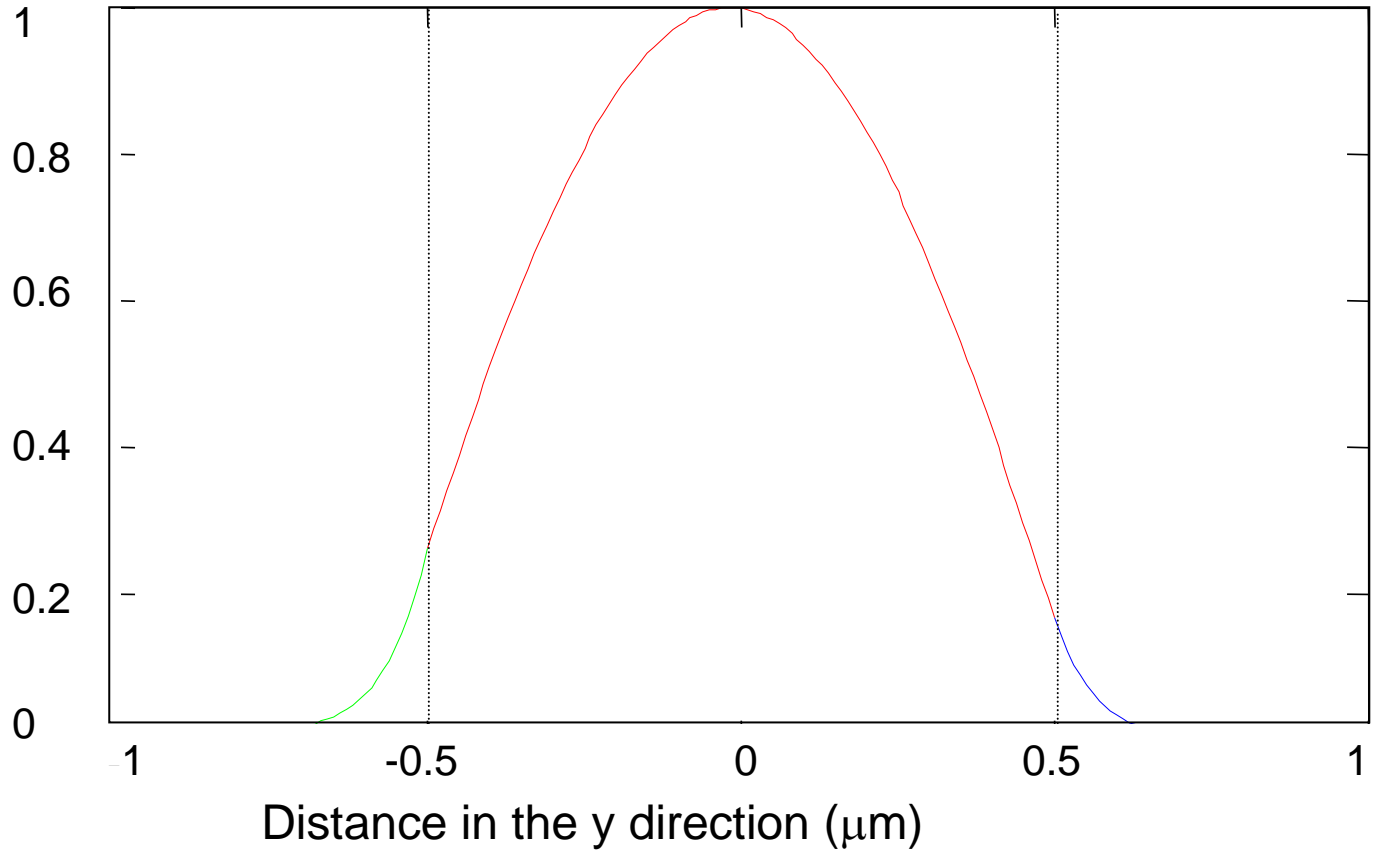
Mode Profiles

Now that we have field solutions for modes in the planar waveguide, we can plot the field distribution, $E_x(y)$ or the intensity distribution, $|E_x(y)|^2$. Consider the following waveguide:

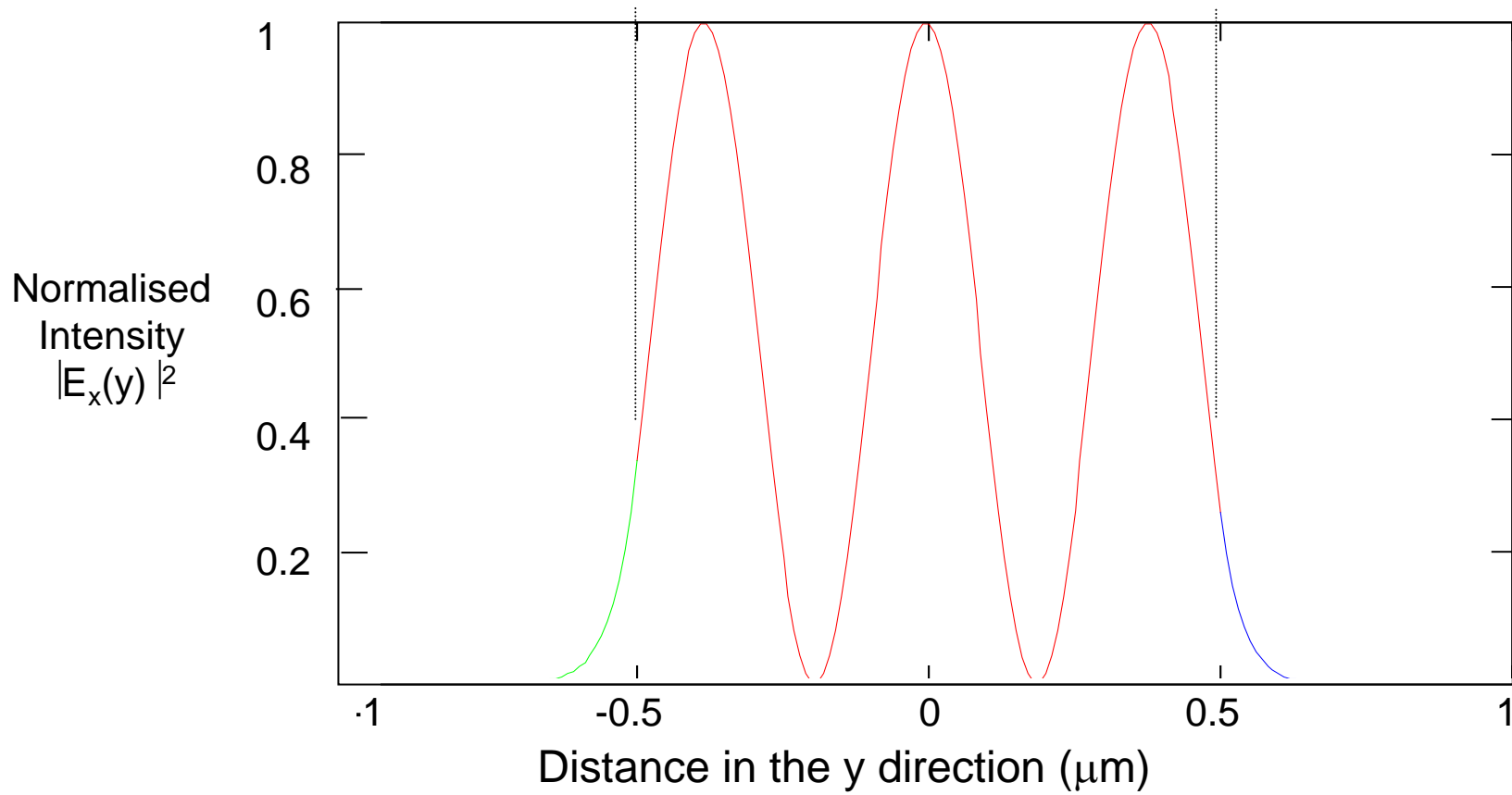
$$n_1 = 3.5, \quad n_2 = 1.5, \quad n_3 = 1.0, \quad \lambda_0 = 1.3\mu\text{m}, \quad \text{and } h = 1.0\mu\text{m}.$$

For even functions, the field in the core is a cosine function, and in the claddings, an exponential decay. Therefore, solutions for $m=0$, and $m=2$ are even functions, and are plotted below. The field distribution is plotted for $m = 0$, and the intensity distribution is plotted for $m = 2$.

Normalised
Electric field
 $E_x(y)$



Electric field profile of the fundamental mode ($m=0$)



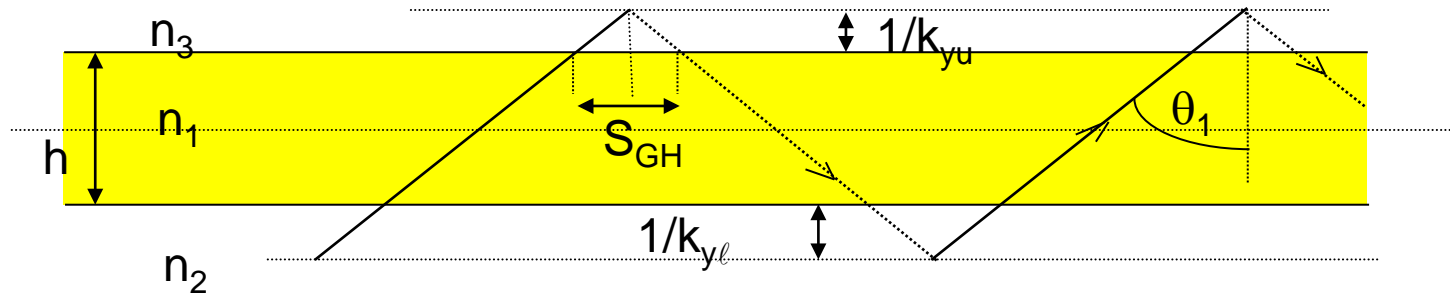
Intensity profile of the 2nd even mode (m = 2)

Confinement factor

How much of the power propagates inside the core? We can define a confinement factor Γ :

$$\Gamma = \frac{\int_{-h/2}^{h/2} \mathbf{E}_x^2(\mathbf{y}) d\mathbf{y}}{\int_{-\infty}^{\infty} \mathbf{E}_x^2(\mathbf{y}) d\mathbf{y}}$$

The Goos-Hänchen Shift



Propagation in an asymmetric planar waveguide showing cladding penetration

This diagram allows the ray model to show a phase change on reflection and cladding penetration, but introduces the lateral shift S_{GH} . By trigonometry, for the upper phase shift:

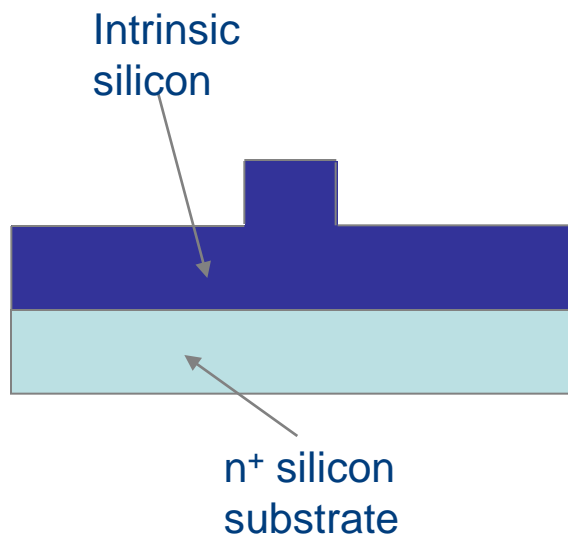
$$\tan \theta_1 = \frac{\frac{S_{GH}}{2}}{\frac{1}{k_{yu}}} = \frac{S_{GH} k_{yu}}{2}$$

$$\text{i.e. } S_{GH} = \frac{2 \tan \theta_1}{k_{yu}} = \frac{2}{k_{yu}} \frac{\beta}{k_{yc}}$$

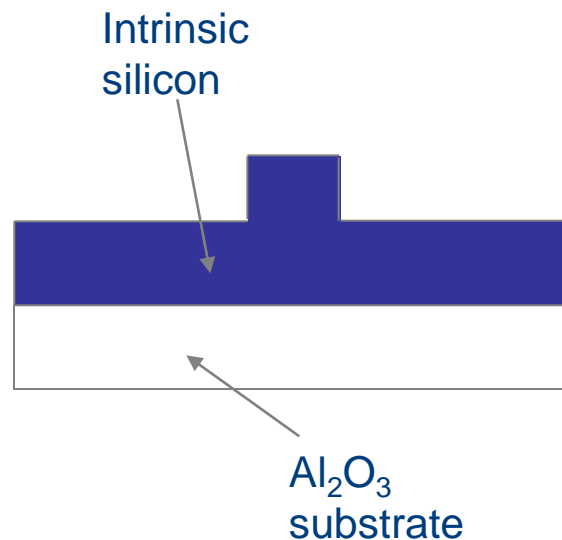
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Options for Silicon waveguides:

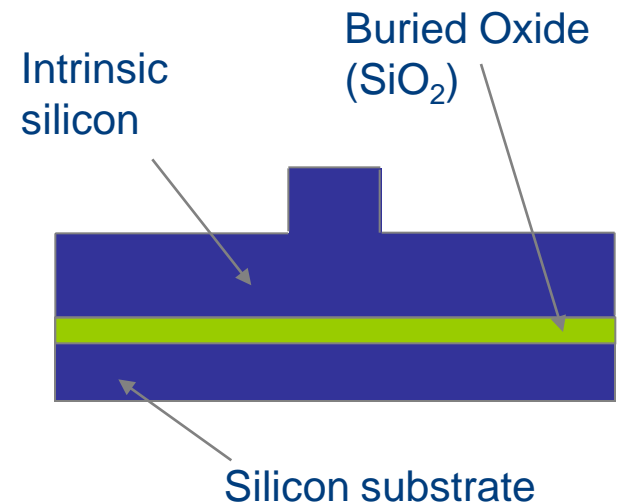
Silicon on doped silicon



Silicon on sapphire



Silicon on Insulator (SOI)

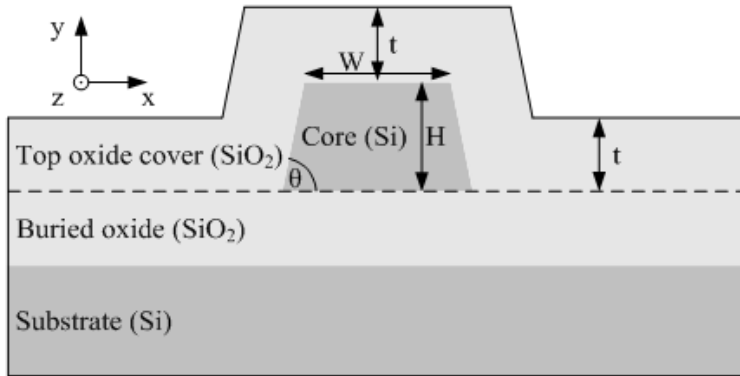


First reported by Soref & Lorenzo
Electron. Lett., 21, 1985. &
IEEE J. Quantum Elect., QE-22,
 1986.

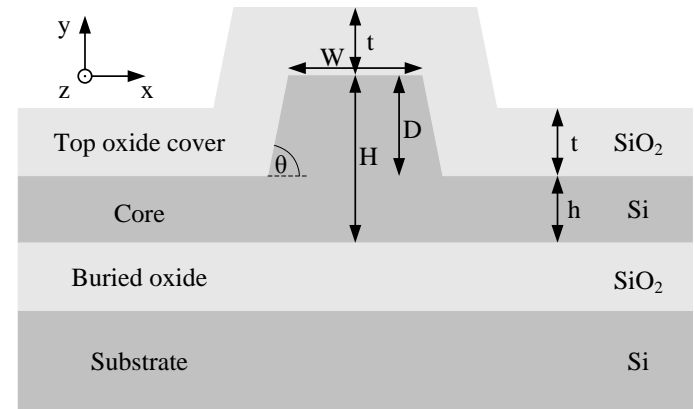
First reported by Albares & Soref
Proc. SPIE, 704, 1987.

First reported by Soref & Lorenzo,
*OSA Integrated Guided Wave
 Optics '89*, 1989.

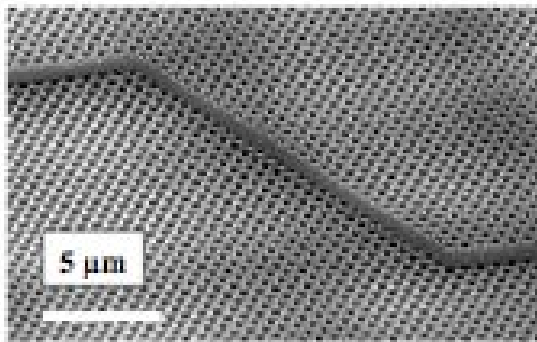
The most popular waveguides



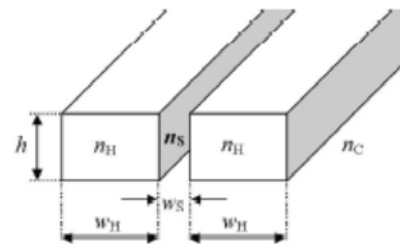
Strip waveguide (SM: 200x500 nm;
2-3 dB/cm loss)



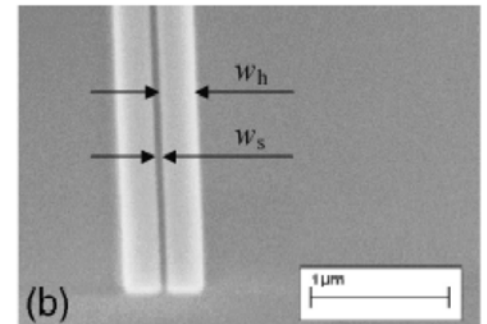
Rib/ridge waveguide (H=200-400 nm
to several microns; 0.1 dB/cm loss)



Photonic crystal waveguide
(L=100 μm; 3-4 dB/cm loss)

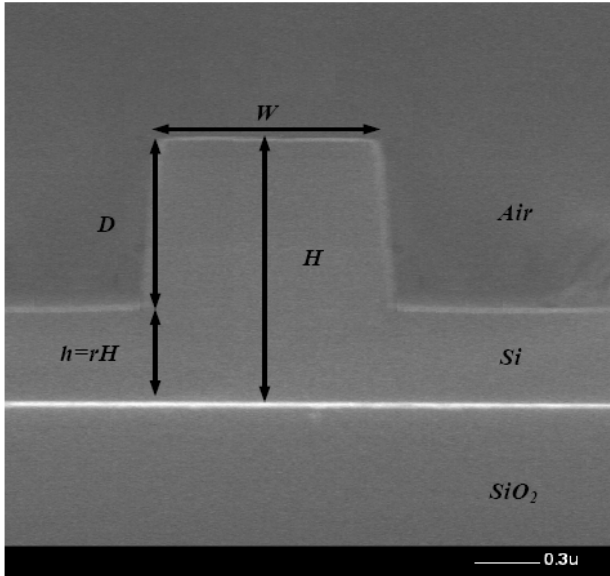


(a)

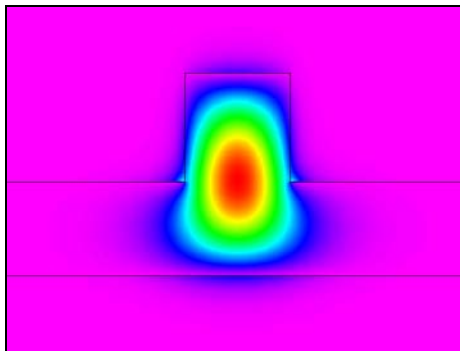


(b)

Slot waveguide (non-linear effects, sensors)

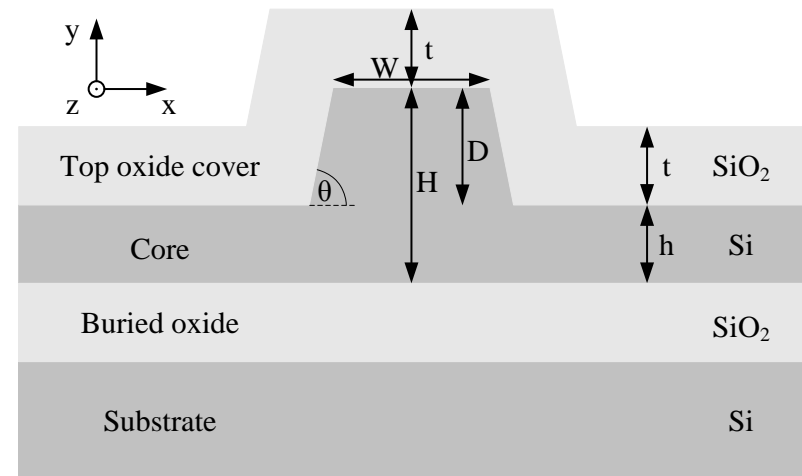


SOI rib waveguide



$$\frac{W}{H} \leq 0.3 + \frac{r}{\sqrt{1-r^2}} \quad (\text{for } 0.5 \leq r < 1)$$

Soref's formula for single mode condition for large rib waveguides



Rib waveguide (H=400 nm to several microns; 0.1 dB/cm loss)

Modes of two dimensional waveguides

Modes are designated

$$\mathbf{E}_{p,q}^x \quad \mathbf{E}_{p,q}^y$$

or

$$\mathbf{HE}_{p,q} \quad \mathbf{EH}_{p,q}$$

p maxima in the x direction, q maxima in the y direction

Hence Fundamental mode:

$$\mathbf{E}_{1,1}^x \quad \text{or} \quad \mathbf{E}_{1,1}^y$$

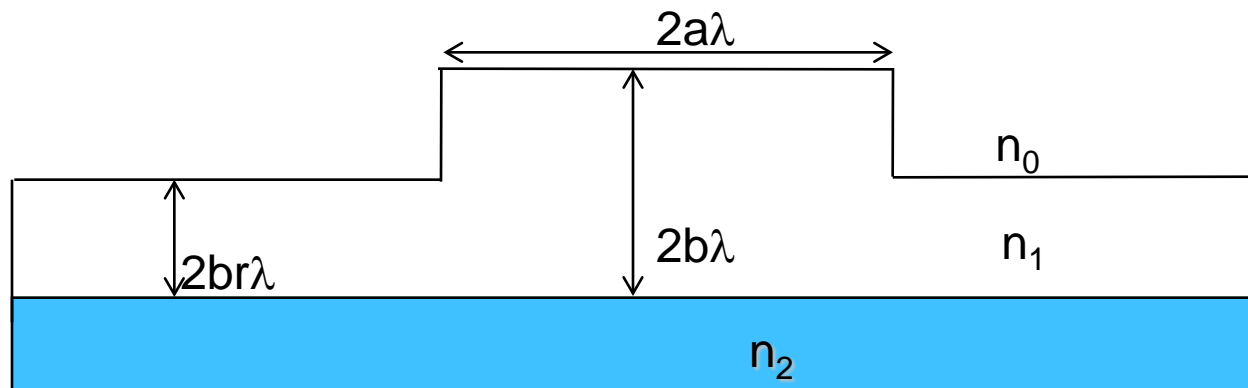
Sometimes labelled

$$\mathbf{E}_{0,0}^x \quad \mathbf{E}_{0,0}^y$$

It is a little surprising that rib waveguides with cross sectional dimensions of several microns are routinely described as single mode waveguides.

The answer to this apparent paradox is that, if the waveguide is correctly designed, higher order modes leak out of the waveguide over a very short distance, as demonstrated by Soref et al, in 1991.

Firstly the authors defined a rib waveguide in terms of some normalising parameters:



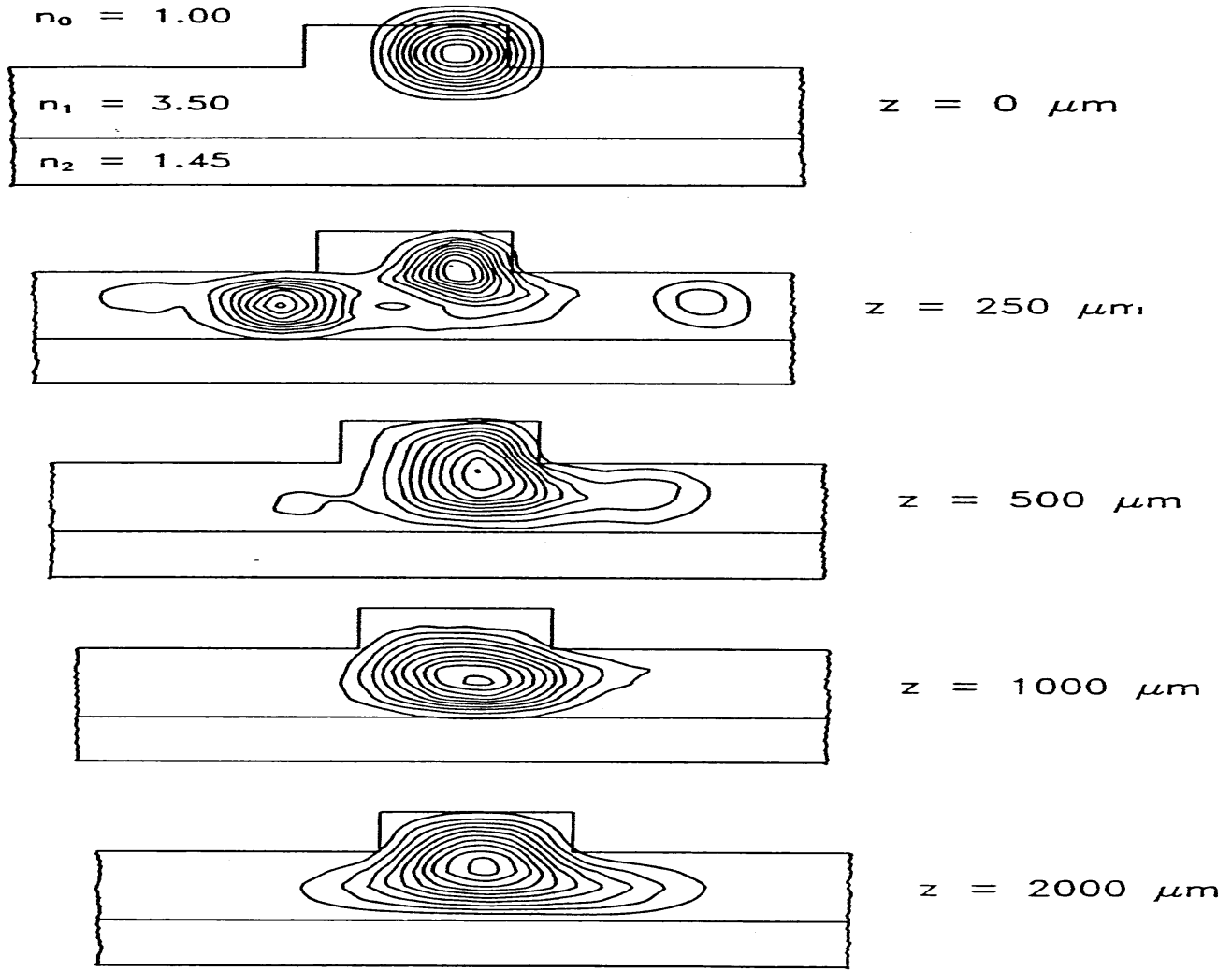
Rib waveguide definitions

The authors limited their analysis to waveguides in which $0.5 \leq r < 1.0$, because for $r \geq 0.5$, the effective index of ‘vertical modes’ in the planar region either side of the rib, becomes higher than the effective index of all vertical modes in the rib, other than the fundamental.

They then used an effective index approach to define a parameter related to the aspect ratio of the rib waveguide, a/b . They then found the limiting condition such that the EH_{01} and HE_{01} just failed to be guided. This resulted in a condition for the aspect ratio:

$$\frac{a}{b} \leq 0.3 + \frac{r}{\sqrt{1-r^2}}$$

In order to demonstrate this they simulated excitation of higher order modes and watched them leak out of the waveguide.

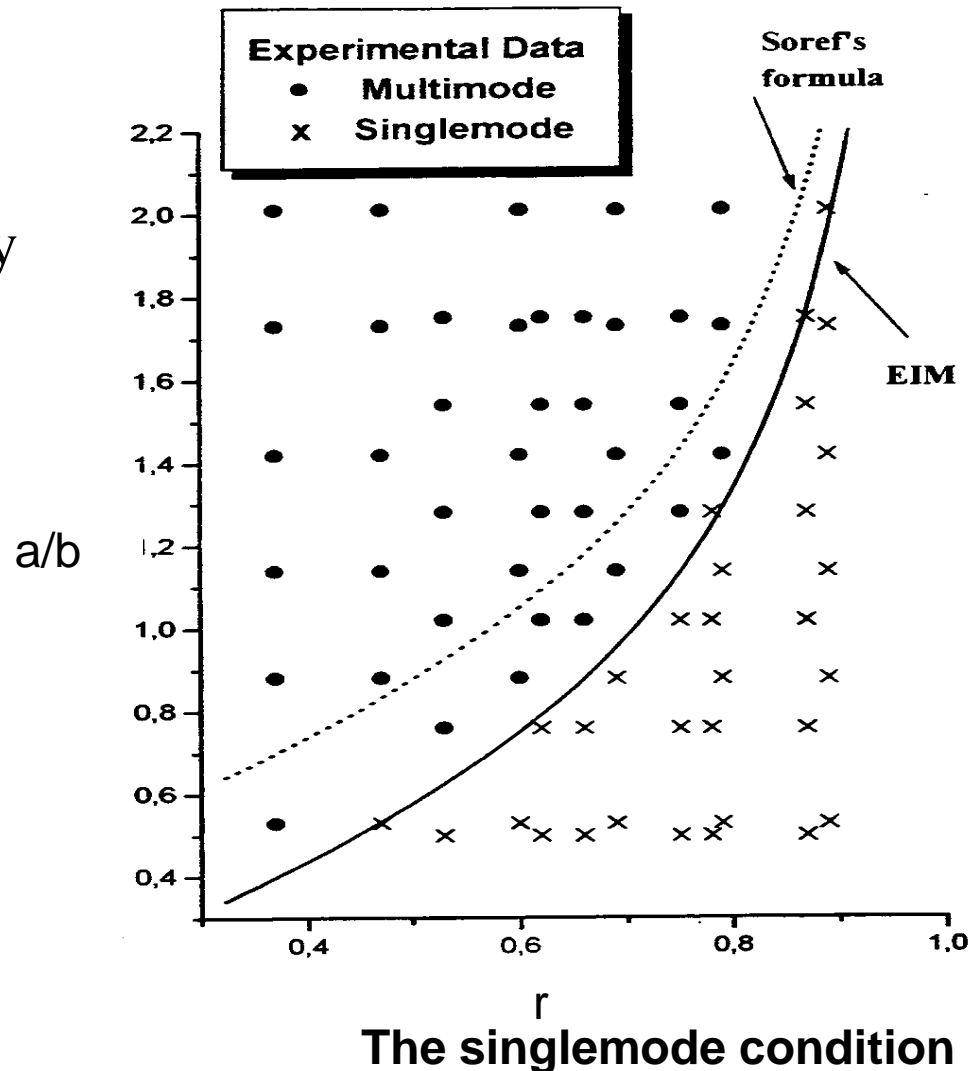


Pogossian et al took the experimental data of Rickman & Reed, and fitted an equation of the form:

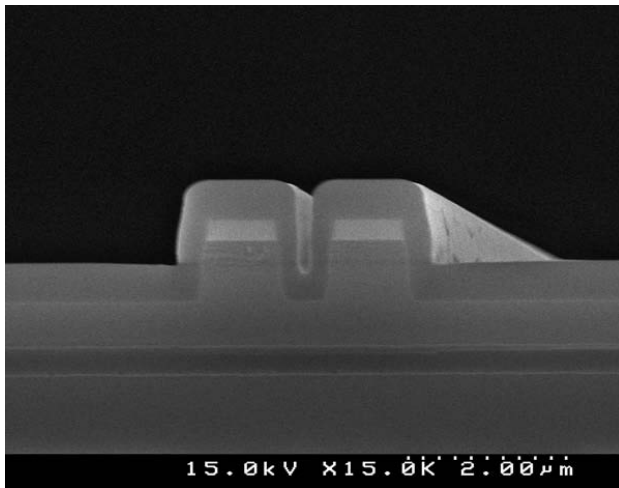
$$\frac{a}{b} \leq c + \frac{r}{\sqrt{1-r^2}}$$

c being approximately 0, resulting in a modified equation:

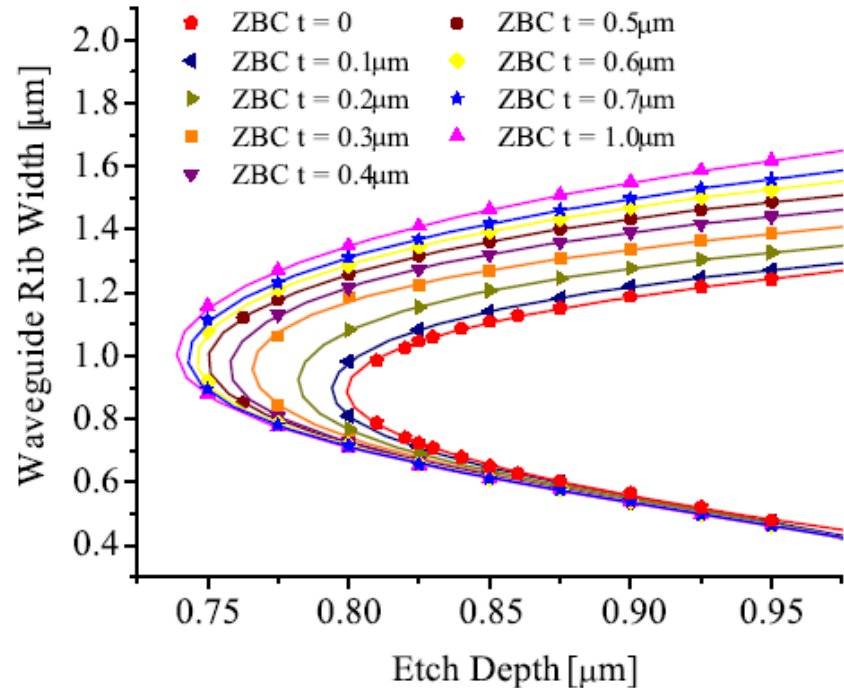
$$\frac{a}{b} \leq \frac{r}{\sqrt{1-r^2}}$$



SOI rib waveguides - modelling

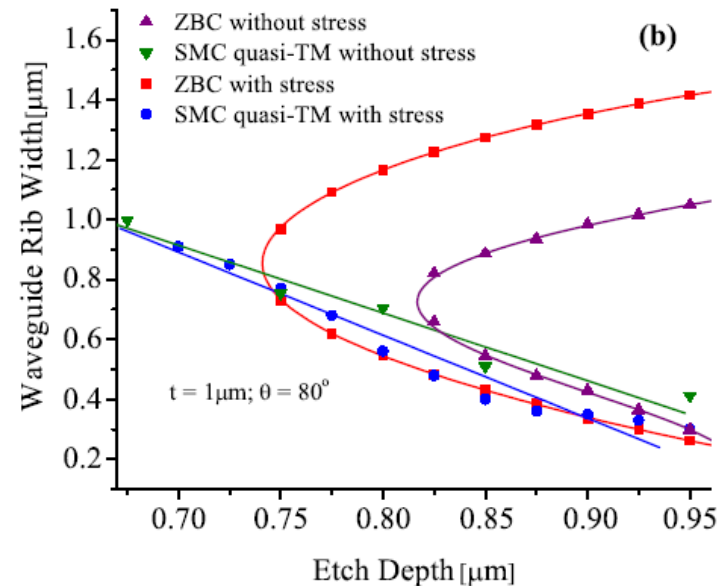
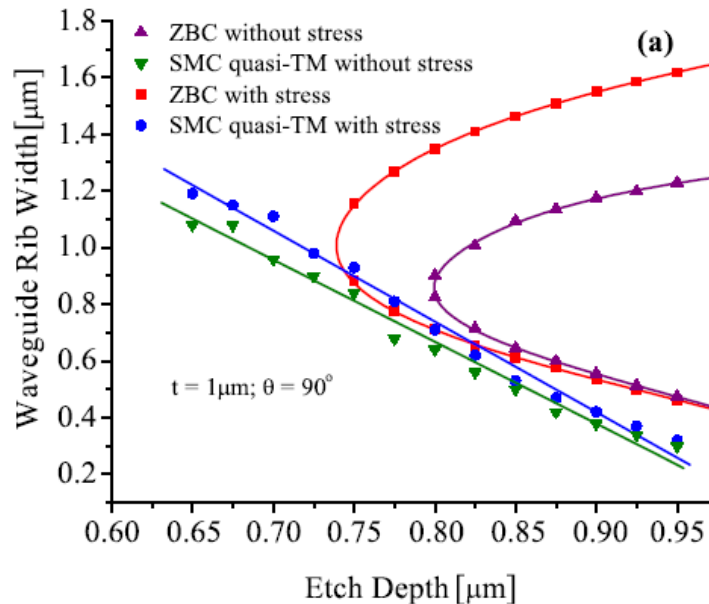


SEM of a directional coupler made of two rib waveguides with oxide cover



The zero-birefringence condition as a function of waveguide rib width and etch depth for different values of top oxide thickness

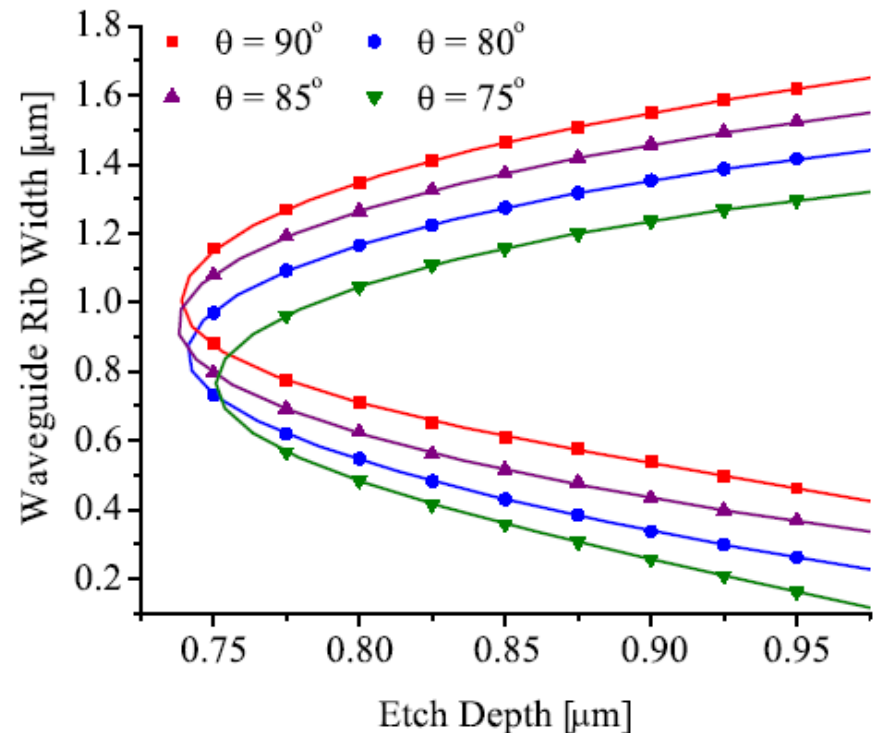
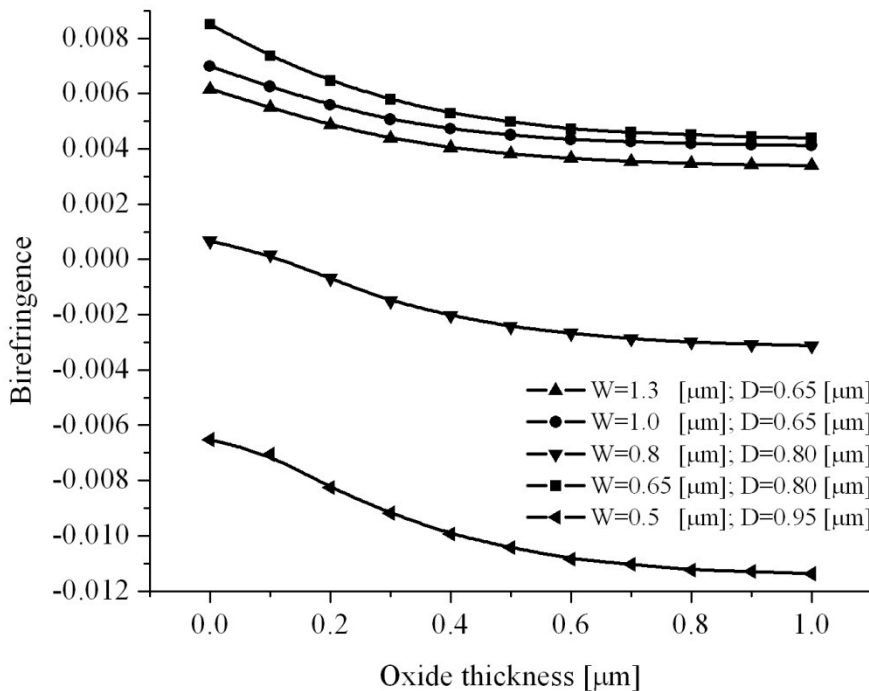
Rib waveguides - modelling



By reducing the side wall angle larger fabrication tolerances can be achieved for fulfilling both zero birefringence (ZB) and single mode (SM) conditions

[M. Milosevic et al., JLT 26 1840 (2008)]

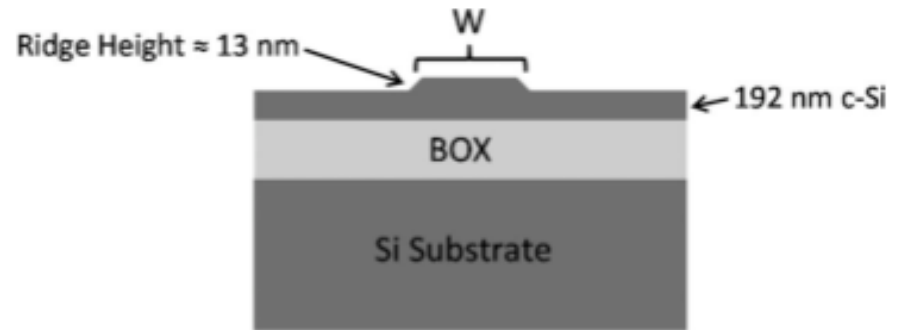
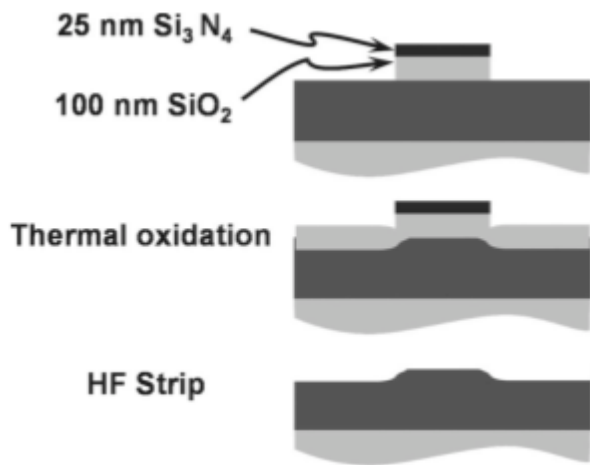
Rib waveguides – oxide thickness and side wall angle influence



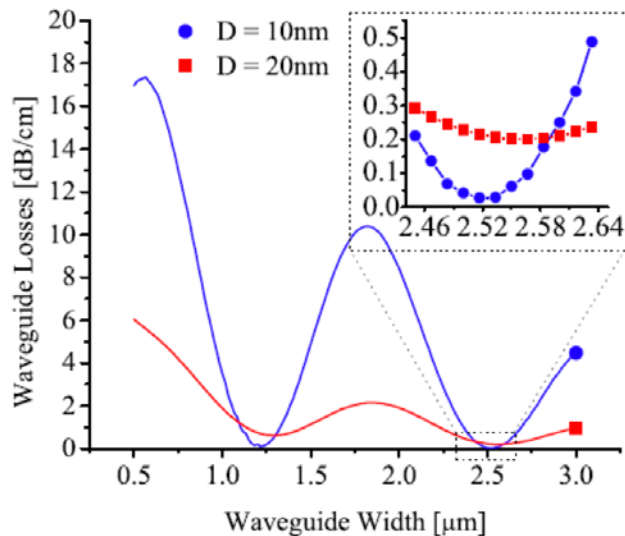
By suitable selection of the waveguide dimensions and the oxide layer parameters, the additional stress can be used to allow greater flexibility in the design of the waveguide [G. Mashanovich et al., SST, 2008]

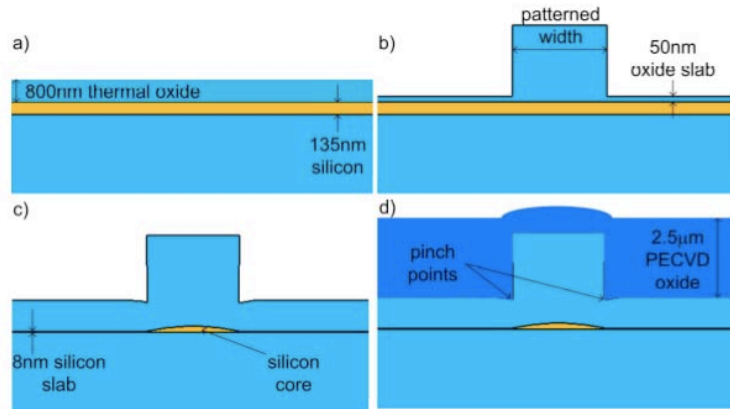
Influence of the sidewall angle on polarisation independent propagation [M. Milosevic et al., JLT 2008]

Low loss etchless waveguides 1

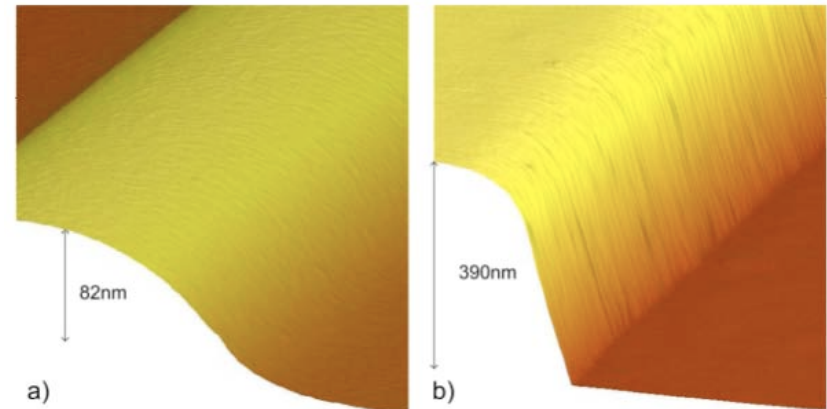
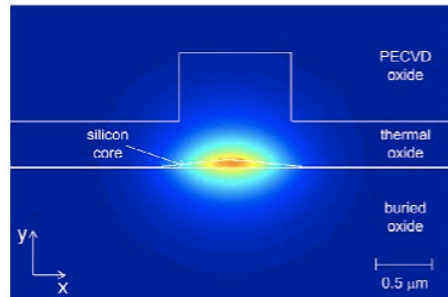


$H=205$ nm, $D=15$ nm, propagation loss 0.36 dB/cm for TE and 0.94 dB/cm for TM polarisation [R. Pafchek et al., Appl. Opt. 48, 958 (2009)]



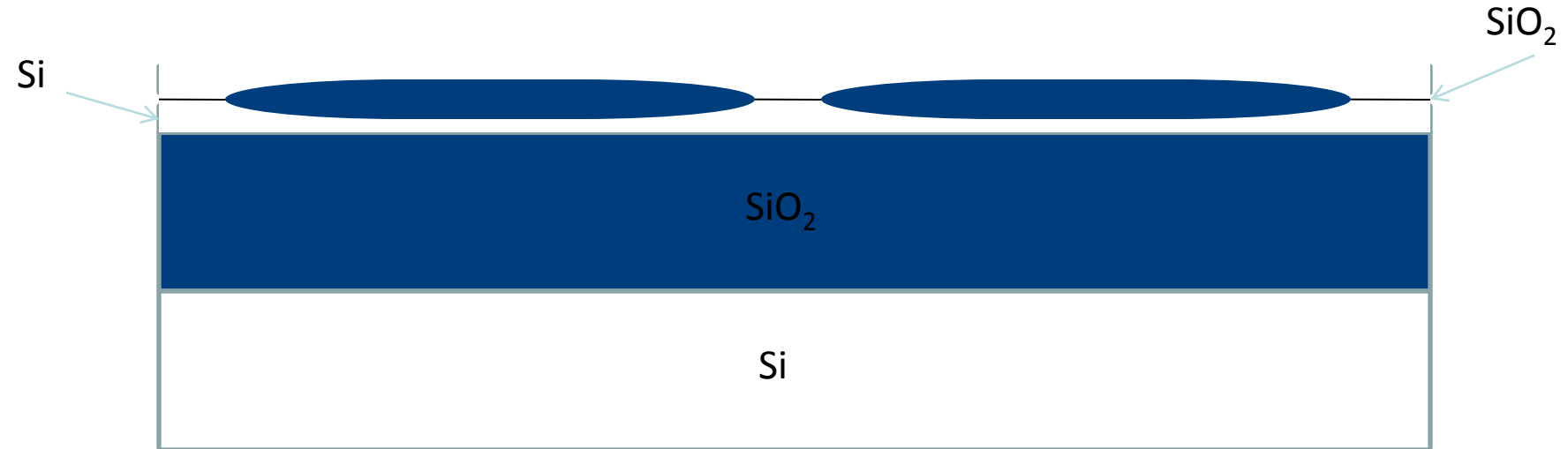


- PECVD oxide
- thermal oxide
- silicon



$H=70$ nm, $W = 1$ μm, loss 0.3 dB/cm, 0.007 dB/bend with $R=50$ μm (0.15 dB/bend with $R=10$ μm) [J. Cardenas et al., OE 17, 4752 (2009)]

LOCAl Oxidation of Silicon (LOCOS) waveguides



Step1 : Oxide deposition PECVD (40nm)

Step2 : Silicon Nitride deposition PECVD (80nm)

Step3 : Silicon Nitride etch

Step4 : Thermal oxidation

Step5 : Removal of nitride and 40nm oxide

Loss as low as 0.1 dB/cm for TE polarisation

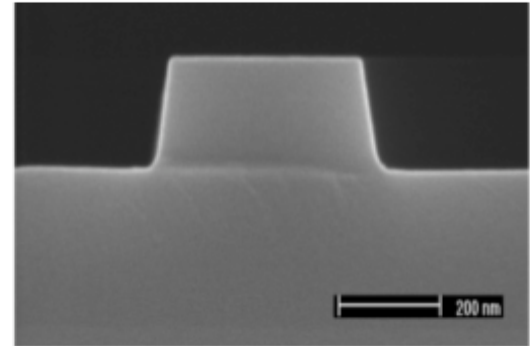
[F. Gardes et al., Photonics West 2008]

Higher losses for TM polarisation

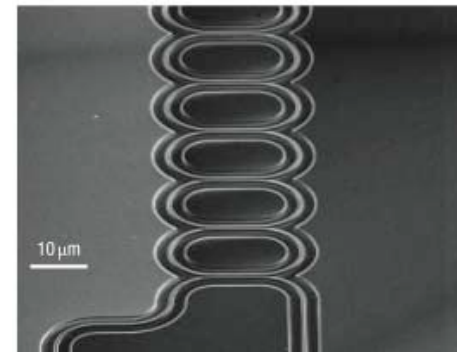
- Waveguide materials
- Planar waveguides
- Rib and ridge waveguides
- **Strip waveguides**
- Losses
- Polarisation issues
- Effect of stress

Strip waveguides / photonic wires

- + Small bending radius
- + Realisation of ultra dense photonic circuits
- + Cost reduction
- + Performance improvement (modulators, filters etc)
- Coupling is problematic
- Higher propagation loss
- Polarisation issues

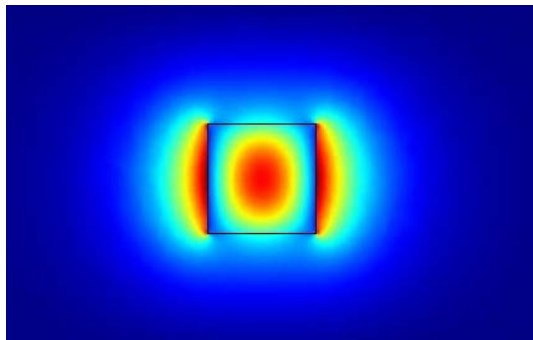
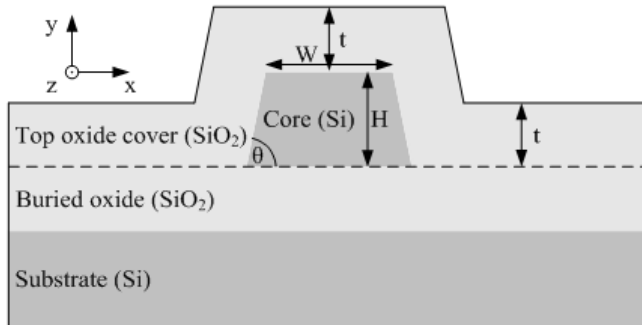


Cross section of a photonic wire



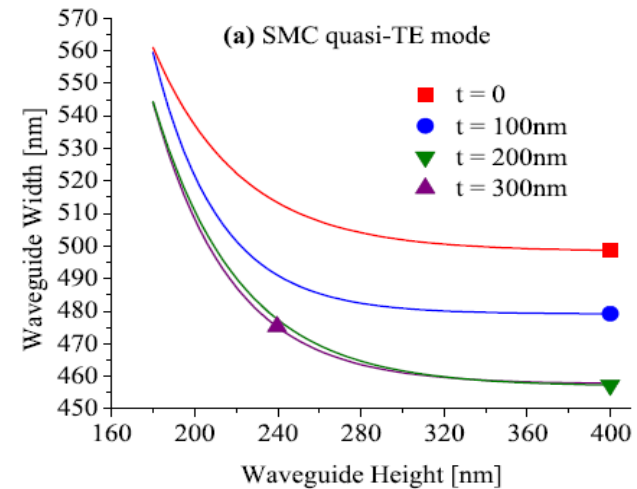
Optical delay line based on silicon photonic wires [F. Xia et al., Nature Photonics 1, 65 (2007)]

SOI photonic wires



Loss:

- 3.6 dB/cm & 0.013 dB/bend ($R=2\mu\text{m}$) for $445\times 220\text{ nm}$ [Vlasov & McNab, OE 12, 1622 (2004)]
- 2.4 dB/cm for $500\times 220\text{ nm}$ with DUV 248nm [P. Dumon et al., PTL 16, 1328 (2004)]
- 2.8 dB/cm for $450\times 220\text{ nm}$ with DUV 193nm [S. K. Selvaraja et al., LEOS Annual meeting 2007]
- 1.7 dB/cm & 0.005 dB/bend ($R=6.5\mu\text{m}$) for $510\times 226\text{ nm}$ [F. N. Xia et al., Nature Photonics 1, 65 (2007)]
- 1 dB/cm for $W=0.5\text{ }\mu\text{m}$ [K. K. Lee et al., OL 26, 1888 (2001)]



Single mode: $W\times H = 200\times 500\text{ nm}$

Other silicon based waveguide materials

Poly-Si: 6.5 dB/cm for 700×250 nm waveguides
[Q. Fang et al., OE 16, 6425 (2008)]

a-Si: 3.5 dB/cm for 480×220 nm waveguides
[S. K. Selvaraja, Opt. Comm. 282, 1767 (2009)]

SiN: 1.5 dB/cm for 310×1050 nm waveguides
[T. Barwicz et al., OE 12, 1437 (2004)]

- Waveguide materials
- Planar waveguides
- Rib and ridge waveguides
- Strip waveguides
- **Losses**
- Polarisation issues
- Effect of stress

Refractive index and loss coefficient in optical waveguides

Complex refractive index can be defined as:

$$n' = n_R + jn_I$$

And we have expressed a propagating field as:

$$\mathbf{E} = \mathbf{E}_0 e^{j(kz - \omega t)}$$

Substituting for propagation constant and complex refractive index:

$$\mathbf{E} = \mathbf{E}_0 e^{j(k_0 n' z - \omega t)} = \mathbf{E}_0 \cdot e^{jk_0 n_R z} \cdot e^{-k_0 n_I z} \cdot e^{-j\omega t}$$

The term $\exp(-k_0 n_I z)$ is often re-designated $\exp(-\frac{1}{2}\alpha z)$. The term α is called the loss coefficient. The factor of $\frac{1}{2}$ is included in the definition above because α is an intensity loss coefficient.

Therefore we can write :

$$\mathbf{I} = \mathbf{I}_0 e^{-\alpha z}$$

The benchmark for acceptable waveguide loss is of the order of 1dB/cm. Typical losses for SOI rib waveguides are in the range 0.1 - 0.5 dB/cm

The contributions to loss in an optical waveguide

Losses result from scattering, absorption, and radiation:

Scattering

Scattering in an optical waveguide can result from two sources: volume scattering and interface scattering

Volume scattering follows either a λ^{-3} dependence, or a λ^{-1} dependence, as a consequence of the type and concentration of scattering centres.

Interface scattering has been modelled by many authors, but a reasonably accurate and attractively simple model was produced by Tien:

$$\alpha_s = \frac{\cos^3 \theta}{2 \sin \theta} \left(\frac{4\pi n_1 (\sigma_u^2 + \sigma_\ell^2)^{\frac{1}{2}}}{\lambda_0} \right)^2 \left(\frac{1}{h + \frac{1}{k_{yu}} + \frac{1}{k_{y\ell}}} \right)$$

where σ_u is the rms roughness for the upper waveguide interface, σ_ℓ is the rms roughness for the lower waveguide interface, k_{yu} is the decay constant in the upper cladding, $k_{y\ell}$ is decay constant in the lower cladding, and h is the waveguide thickness

Example of interface scattering

Consider a planar waveguide:

$n_1 = 3.5$; $n_2 = 1.5$; $n_3 = 1.0$; $h = 1.0\mu\text{m}$, and the operating wavelength $\lambda_0 = 1.3\mu\text{m}$. Let us compare the scattering loss of two different modes of the waveguide, say the TE_0 and the TE_2 modes.

The propagation angles, θ_1 of these two are 80.8° (TE_0), and 60.7° (TE_2). Thus the decay constants in the claddings are given by:

$$k_{yi}^2 = \beta^2 - k_0^2 n_i^2$$

Therefore we evaluate the decay constants for each mode as follows:

TE_0	TE_2
$k_{yu} = \sqrt{\beta^2 - k_0^2 n_3^2} = 15.98 \mu\text{m}^{-1}$	$k_{yu} = \sqrt{\beta^2 - k_0^2 n_3^2} = 13.94 \mu\text{m}^{-1}$
$k_{yl} = \sqrt{\beta^2 - k_0^2 n_2^2} = 15.04 \mu\text{m}^{-1}$	$k_{yl} = \sqrt{\beta^2 - k_0^2 n_2^2} = 12.85 \mu\text{m}^{-1}$

If we now let both σ_u and σ_ℓ equal 1nm, we can evaluate the scattering loss for each mode, for each nanometre of rms roughness at each interface.

For the TE₀ mode,

$$\alpha_s = \frac{\cos^3 \theta}{2 \sin \theta} \left(\frac{4\pi n_1 (\sigma_u^2 + \sigma_\ell^2)^{\frac{1}{2}}}{\lambda_0} \right)^2 \left(\frac{1}{h + \frac{1}{k_{yu}} + \frac{1}{k_{y\ell}}} \right) = 0.04 \text{cm}^{-1}$$

This is equivalent to a loss of 0.18 dB/cm.

For the TE₂ mode

$$\alpha_s = \frac{\cos^3 \theta}{2 \sin \theta} \left(\frac{4\pi n_1 (\sigma_u^2 + \sigma_\ell^2)^{\frac{1}{2}}}{\lambda_0} \right)^2 \left(\frac{1}{h + \frac{1}{k_{yu}} + \frac{1}{k_{y\ell}}} \right) = 1.33 \text{cm}^{-1}$$

This is equivalent to a loss of 5.79 dB/cm.

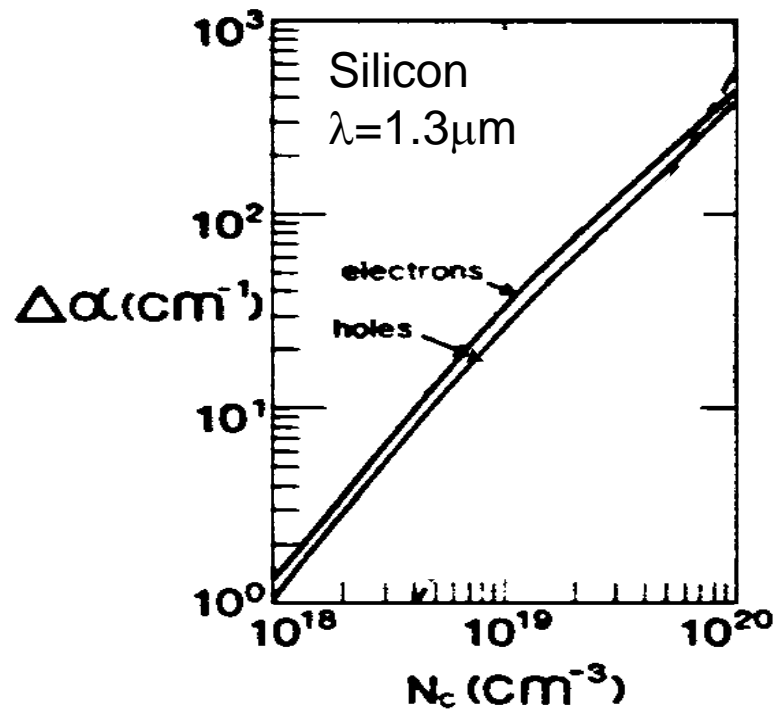
Absorption

The two main potential sources of absorption loss for semiconductor waveguides are band edge absorption and free carrier absorption. If we operate at a wavelength well away from the band edge, the former is negligible.

Changes in free carrier absorption can be described by Drude-Lorenz equation:

$$\Delta\alpha = \frac{e^3\lambda_0^2}{4\pi^2c^3\varepsilon_0n} \left(\frac{N_e}{\mu_e(m_{ce}^*)^2} + \frac{N_h}{\mu_h(m_{ch}^*)^2} \right)$$

where e is the electronic charge; c is the velocity of light in vacuum; μ_e is the electron mobility; μ_h is the hole mobility; m_{ce}^* is the effective mass of electrons; m_{ch}^* is the effective mass of holes; N_e is the free electron concentration; N_h is the free hole concentration; ε_0 is the permittivity of free space; and λ_0 is the free space wavelength.



10^{18} cm^{-3} electrons and holes introduces an additional loss of $\sim 2.5 \text{ cm}^{-1}$. i.e 10.86 dB/cm!

Additional loss of silicon due to free carriers

Radiation Losses in optical waveguides

Ideally negligible.

Possibility of radiation via leaky modes or curvature at too fast a rate.

Reflection from the waveguide facet

Reflection coefficient for TE polarisation (using the Fresnel equations):

$$r_{\text{TE}} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

Similarly, the reflection coefficient r_{TM} was:

$$r_{\text{TM}} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

Using Snells Law r_{TE} reduces to:

$$r_{\text{TE}} = \frac{-\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

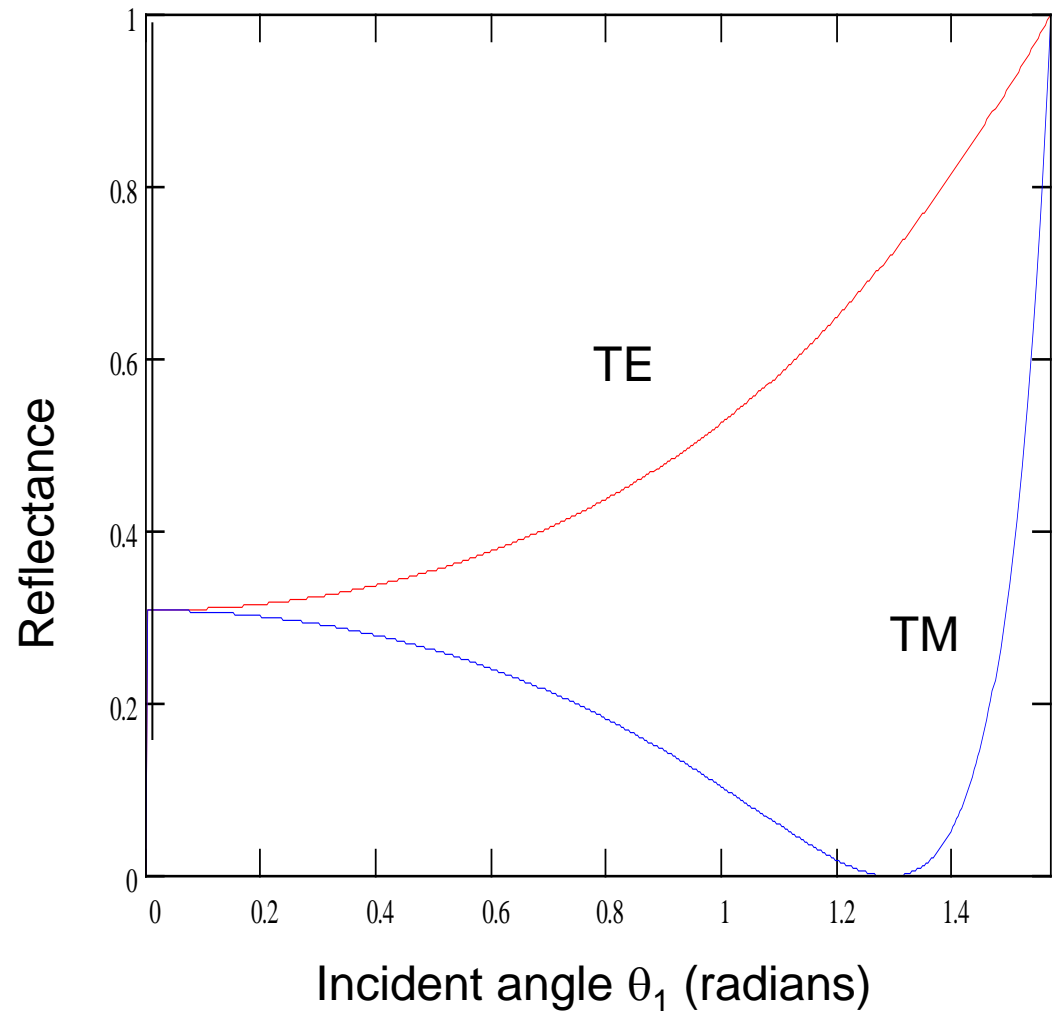
Hence the reflectivity R is given by:

$$R_{\text{TE}} = r_{\text{TE}}^2 = \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}$$

Similarly R_{TM} can be found to be

$$R_{\text{TM}} = r_{\text{TM}}^2 = \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}$$

The two functions are plotted for an air/silicon interface (i.e. $n_1 = 1.0$, $n_2 = 3.5$).



Reflection at an air/silicon interface

At normal incidence ($\theta_1=0$), the reflection of both TE and TM polarisations is the same. Furthermore, end-fire coupling introduces light at near normal incidence. Consequently the approximation is usually made that the Fresnel reflection at the waveguide facets is that due to normal incidence. In this case, reflectivity is:

$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$$

For a silicon/air interface this reflection is approximately 31%, which introduces an additional loss of 1.6dBs. A loss of 1.6dBs for each facet of the waveguide is considerable, and is reduced in commercial devices by the use of anti-reflection coatings.

An anti-reflection coating has a thickness of $\lambda/4$, reducing or eliminating the reflection. For normal incidence, the net reflectivity R is given by:

$$R = \left| \frac{n_1 n_2 - n_{\text{ar}}^2}{n_1 n_2 + n_{\text{ar}}^2} \right|^2$$

where n_{ar} is the refractive index of the anti-reflection coating. R will be zero if:

$$n_1 n_2 = n_{\text{ar}}^2$$

For a silicon/air interface, n_{ar} needs to be approximately 1.87.

For silicon nitride (Si_3N_4), $n = 2.05$.

For silicon oxynitride (SiO_xN_y), n ranges from 1.46 (SiO_2) to 2.05 (Si_3N_4)

Measurement of propagation loss in integrated optical waveguides

There is often confusion between insertion loss and propagation loss

Insertion loss and propagation loss

The insertion loss of a device, is the total loss associated with introducing that element into a system, and includes the inherent loss and the coupling losses.

Alternatively, the propagation loss is the loss associated with propagation in the waveguide alone – i.e. measurement of loss coefficient, α .

There are three main experimental techniques associated with waveguide measurement. These are (i) the cut-back method; (ii) the Fabry-Perot resonance method; and (iii) scattered light measurement.

The cut-back method

The cut back method is conceptually simple. A waveguide of length L_1 is excited by one of the coupling methods mentioned, and the output power from the waveguide, I_1 , and the input power to the waveguide, I_0 are recorded. The waveguide is then shortened to another length, L_2 , and the measurement repeated to determine I_2 . Hence:

$$\frac{I_1}{I_2} = \exp(-\alpha(L_1 - L_2))$$

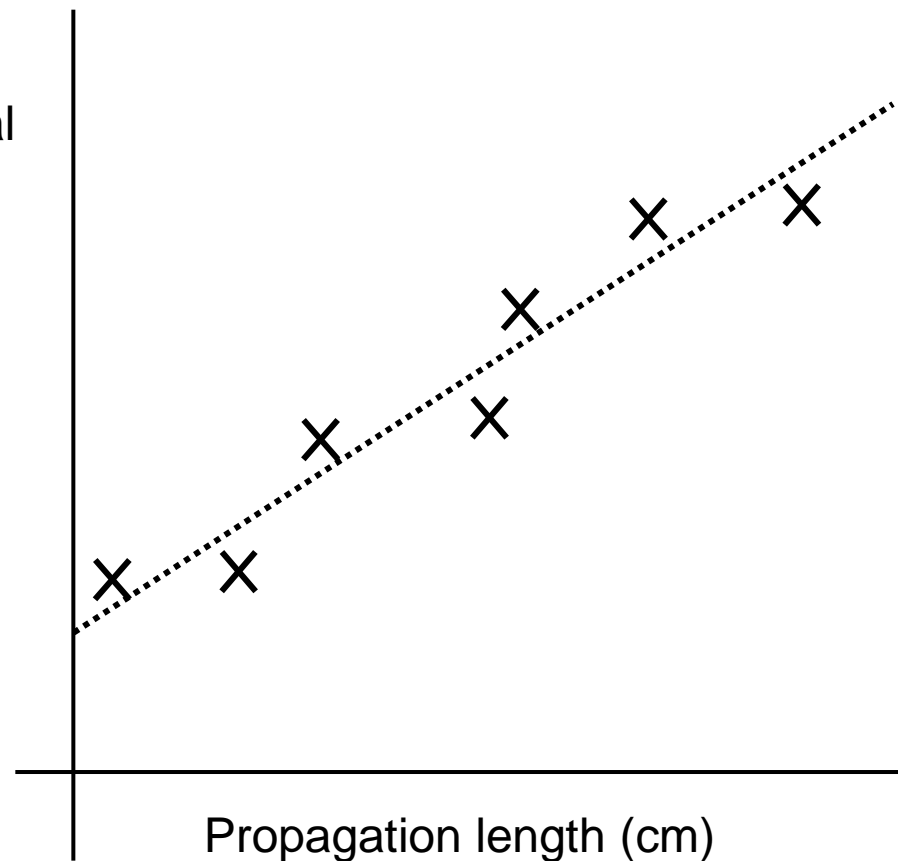
i.e.

$$\alpha = \left(\frac{1}{L_1 - L_2} \right) \ln \left[\frac{I_2}{I_1} \right]$$

The accuracy of the technique can be improved by taking multiple measurements and plotting a graph

Note that the data presented is now insertion loss for each device length, not propagation loss. Hence the loss at zero propagation loss represents coupling loss.

Optical Loss (dBs)



A useful variation of the cut-back method, is to carry out an insertion loss measurement for a single waveguide length, and calculate the coupling. Whilst this technique is less, it has the enormous advantage of being non-destructive

The Fabry-Perot resonance method

An optical waveguide with polished end faces (facets), is similar in structure to the cavity of a laser, and may be regarded as a potentially resonant cavity. Such a cavity is called a Fabry-Perot cavity. The optical intensity transmitted through such a cavity, I_t , is related to the incident light intensity, I_0 , by the well known equation:

$$\frac{I_t}{I_0} = \frac{(1 - R)^2 e^{-\alpha L}}{(1 - R e^{-\alpha L})^2 + 4 R e^{-\alpha L} \sin^2\left(\frac{\phi}{2}\right)}$$

where R is the facet reflectivity, L is the waveguide length, α is the loss coefficient, and ϕ is the phase difference between successive waves in the cavity

This transfer function has a maximum value when $\phi=0$ (or multiples of 2π), and a minimum value when $\phi=\pi$. i.e. :

$$\frac{I_{\max}}{I_0} = \frac{(1-R)^2 e^{-\alpha L}}{(1 - R e^{-\alpha L})^2}$$

$$\frac{I_{\min}}{I_0} = \frac{(1-R)^2 e^{-\alpha L}}{(1 - R e^{-\alpha L})^2 + 4 R e^{-\alpha L}} = \frac{(1-R)^2 e^{-\alpha L}}{(1 + R e^{-\alpha L})^2}$$

Therefore the ratio of the maximum intensity to minimum intensity,

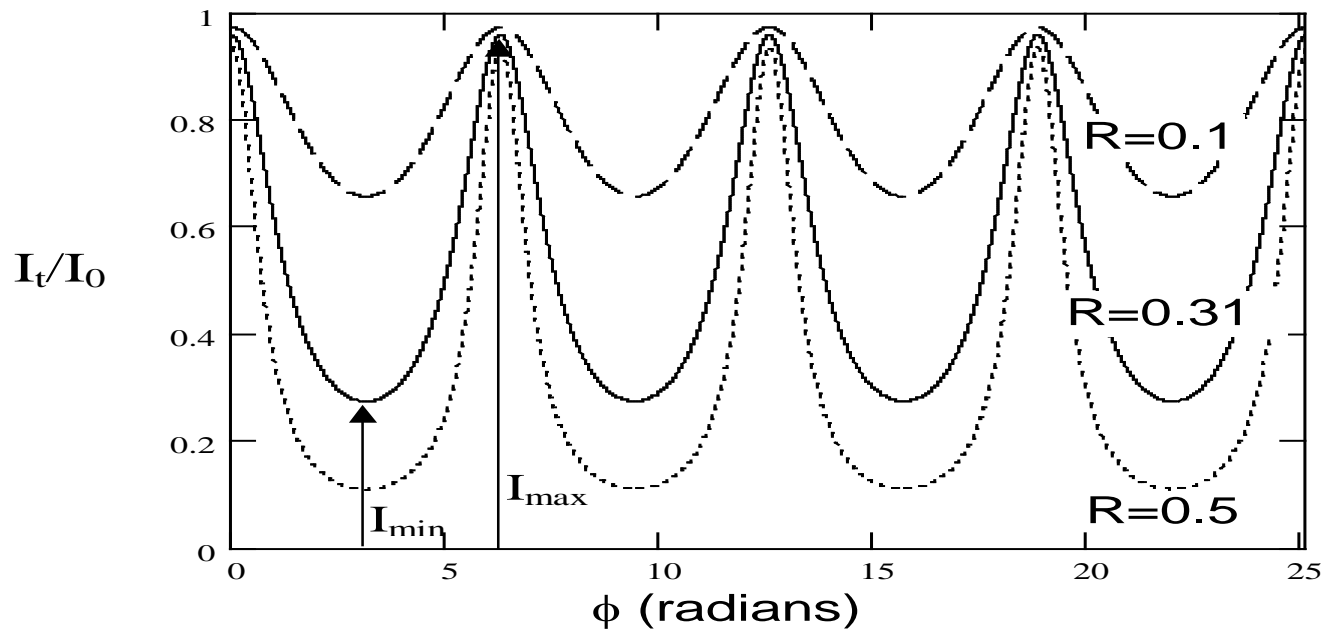
ζ , is:

$$\zeta = \frac{I_{\max}}{I_{\min}} = \frac{(1 + R e^{-\alpha L})^2}{(1 - R e^{-\alpha L})^2}$$

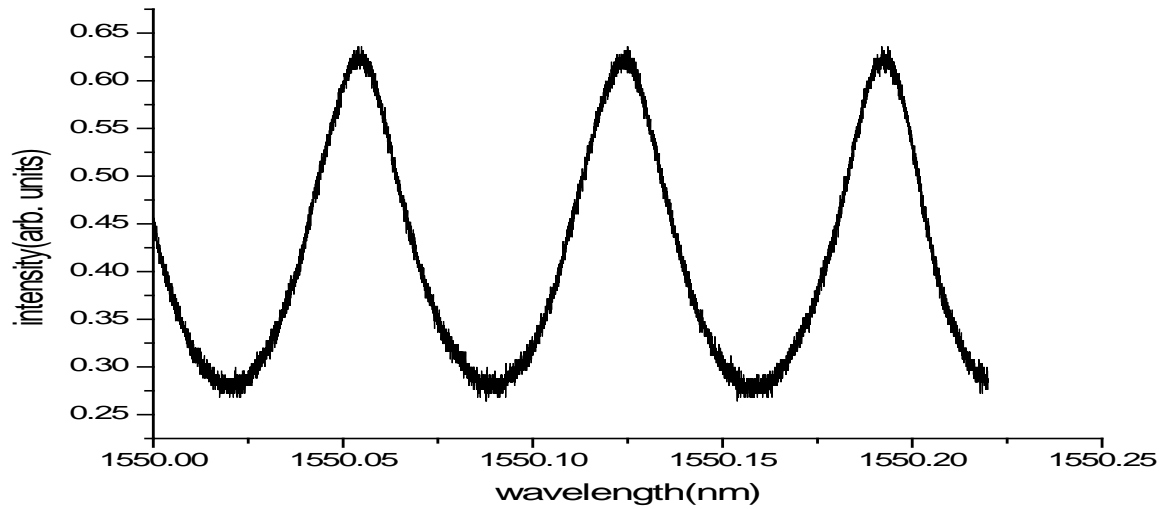
Which we can rearrange as:

$$\alpha = -\frac{1}{L} \ln \left[\frac{1}{R} \frac{\sqrt{\zeta} - 1}{\sqrt{\zeta} + 1} \right]$$

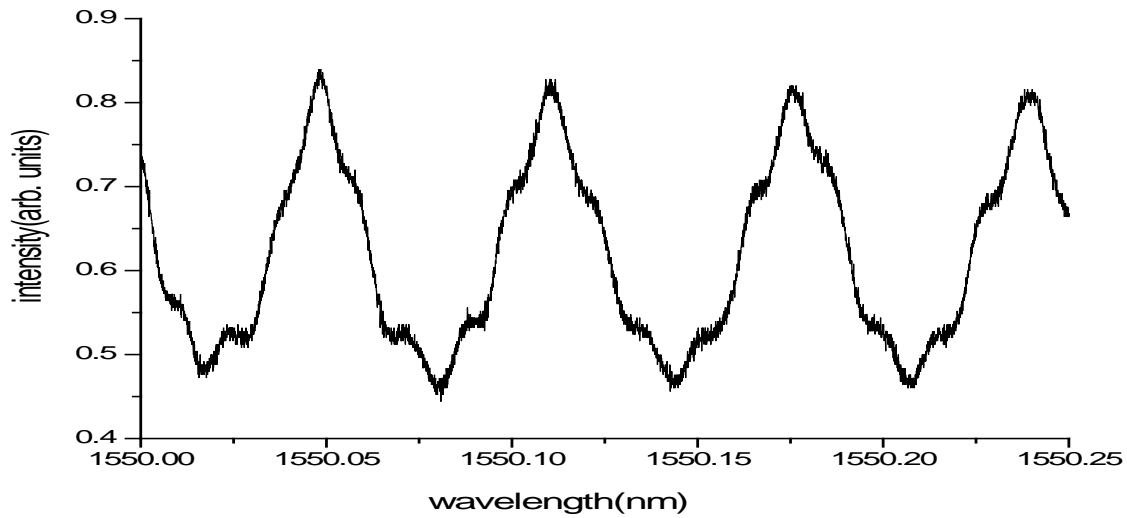
If we know the reflectivity, R , if we can measure ζ , the loss coefficient can be evaluated. Therefore if we can sweep through a few cycles of 2π , ζ can be measured. Such cycling can be achieved thermally, or by varying the wavelength of the light source.



Plot of the Fabry-Perot transfer function for three different mirror reflectivities



Fabry-Perot scan of a single mode waveguide



Fabry-Perot scan of a multimode waveguide

Scattered Light Measurement

The measurement of scattered light from the surface of a waveguide can be used to determine the loss. The assumption is that the amount of light scattered is proportional to the propagating light. Therefore, the rate of decay of scattered light with length will mimic the rate of decay of light in the waveguide.

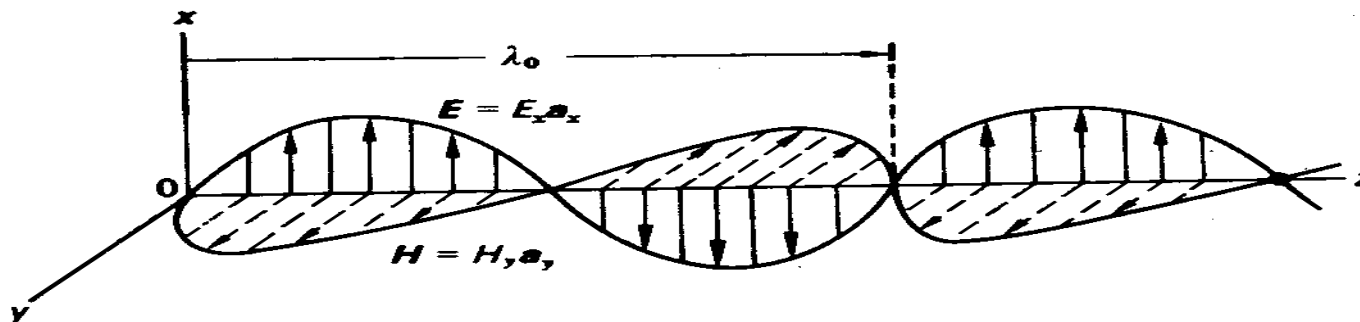
However, it is clear that light is only scattered significantly if the loss of the waveguide is high, so this approach has limited uses.

- Waveguide materials
- Planar waveguides
- Rib and ridge waveguides
- Strip waveguides
- Losses
- **Polarisation issues**
- Effect of stress

What is polarisation?

Light is an electromagnetic wave, which has the characteristic of both electric and magnetic fields that vary with time.

Consider a sinusoidal plane wave:



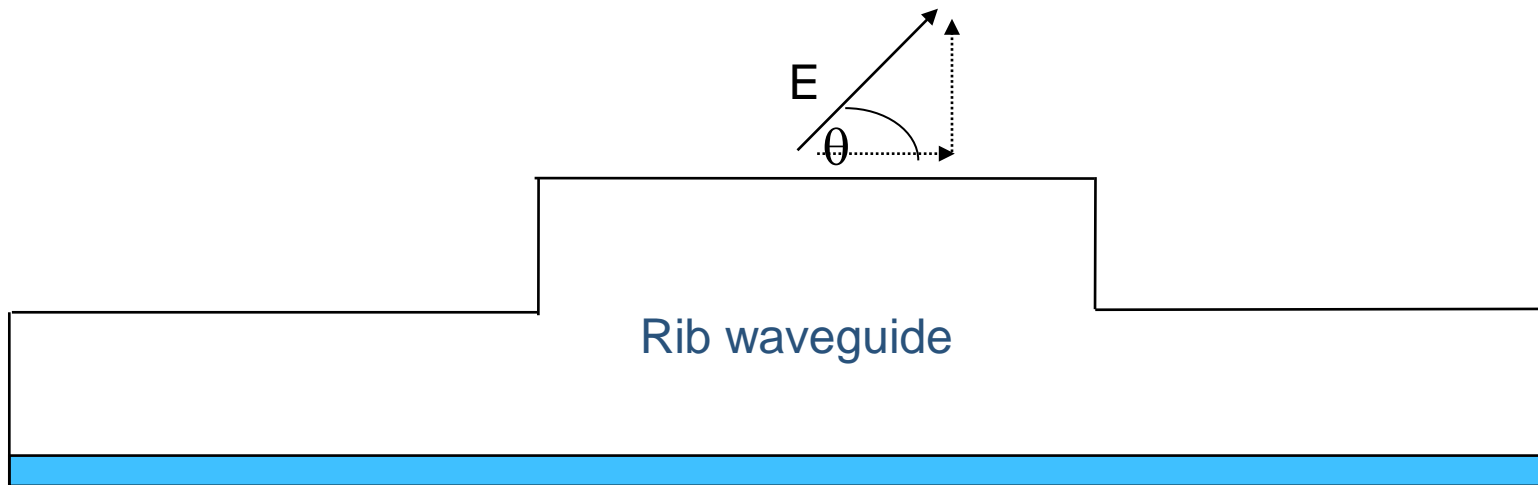
Sinusoidal plane wave showing electric and magnetic fields

This figure shows 3 characteristics:

1. The wave is a plane wave because the value of the electric field (shown in the x direction) and the magnetic field (shown in the y direction), are constant in any z-plane.
2. Secondly, the wave is transverse because both the electric and magnetic fields are transverse to the direction of propagation (z direction).
3. The wave is **polarised**, because the electric and magnetic field exist in only a single direction. This particular wave is polarised in the x direction because the electric field exists only in the x direction.

There are various forms of polarisation, but the most important waves for optical circuits are plane polarised or unpolarised (random).

Light in an optical waveguide propagates in (almost) plane polarised modes, and the plane is either vertical or horizontal with respect to the waveguide surface. These two components can propagate through the circuit with very different characteristics, resulting in entirely different performance.

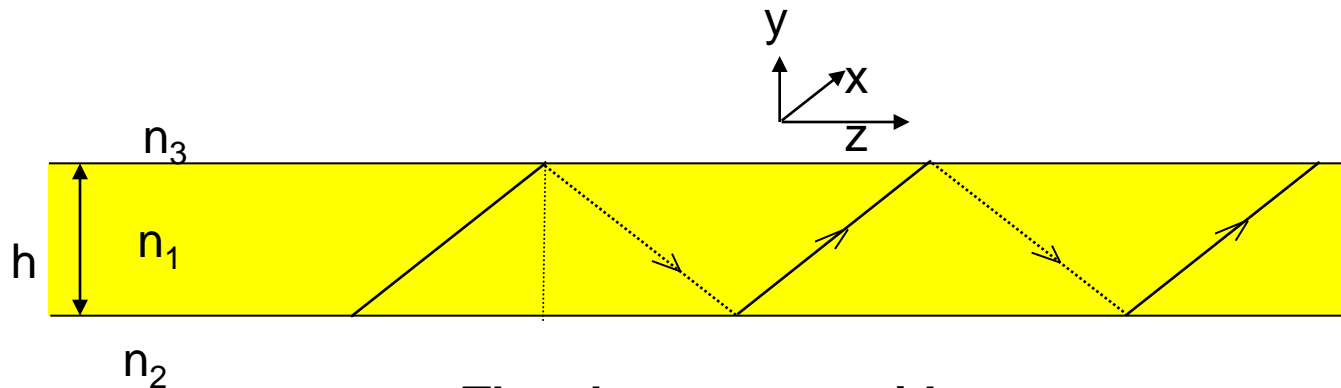


Light polarised at an arbitrary angle θ can be resolved into components parallel and perpendicular to the waveguide surface

The response of the optical circuit to the incoming optical signal should ideally be the same regardless of the polarisation of that optical beam. This is important because the polarisation of the incoming signal may vary, particularly if it originates from a circularly symmetrical optical fibre, which typically provides a signal of random polarisation.

The net result of different responses to the polarisation of the incoming signal usually manifests itself as a different signal loss to each polarisation or a differential phase shift. The difference in loss between the orthogonal polarisation components of the signal is usually termed Polarisation Dependant Loss (PDL).

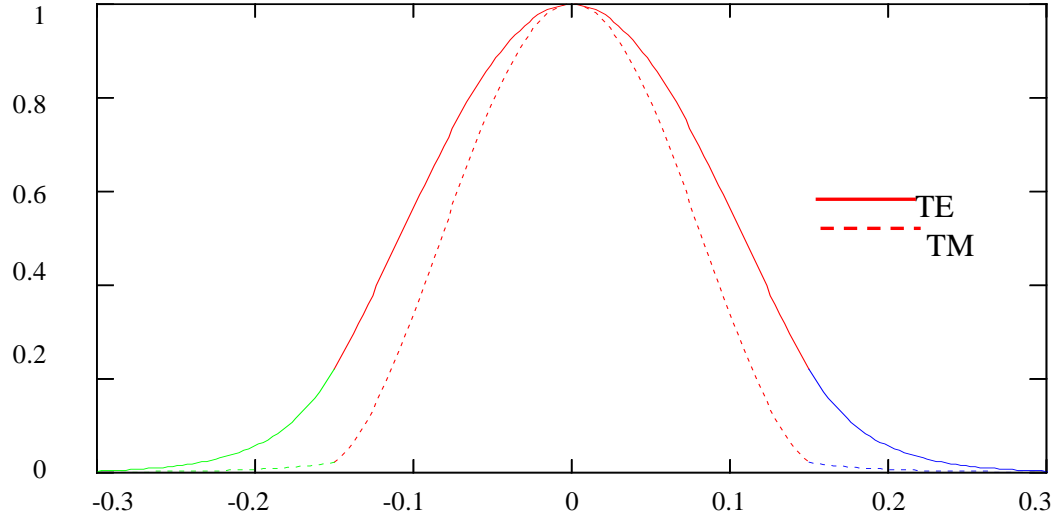
The effect of waveguide thickness



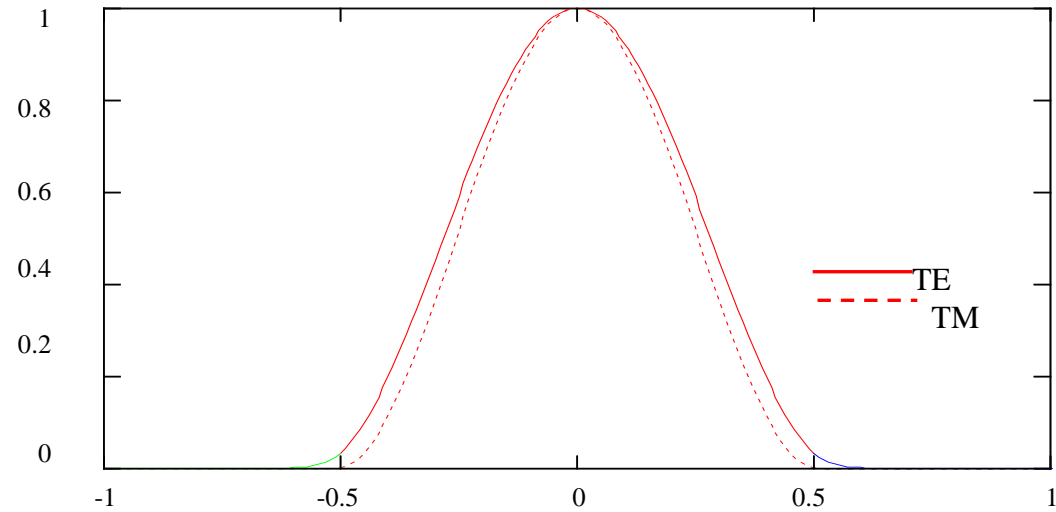
The planar waveguide

Consider the mode profiles of a series of planar waveguides, with the following parameters:

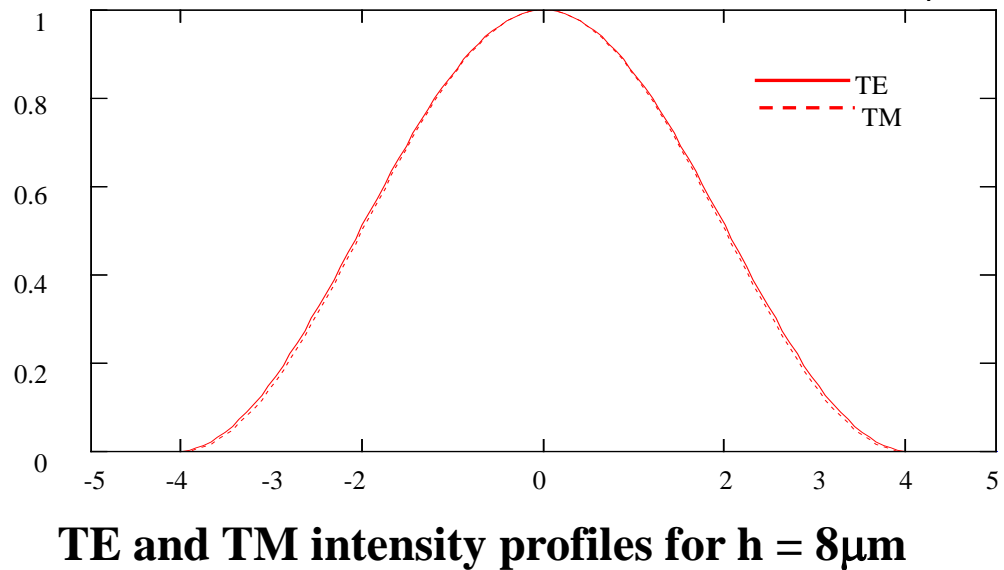
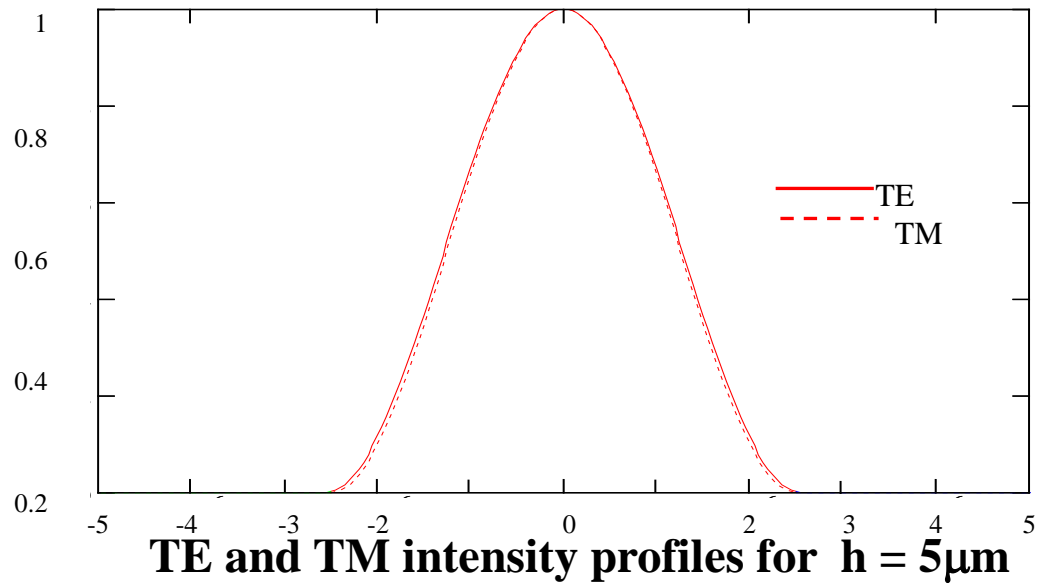
$n_1 = 3.5$ (silicon), $n_2 = n_3 = 1.5$ (silicon dioxide), $\lambda_0 = 1.3\mu\text{m}$, and four waveguide heights of $h = 0.3\mu\text{m}$, $1.0\mu\text{m}$, $5\mu\text{m}$, and $8\mu\text{m}$



Distance in the y direction (μm)
TE and TM intensity profiles for $h = 0.3 \mu\text{m}$



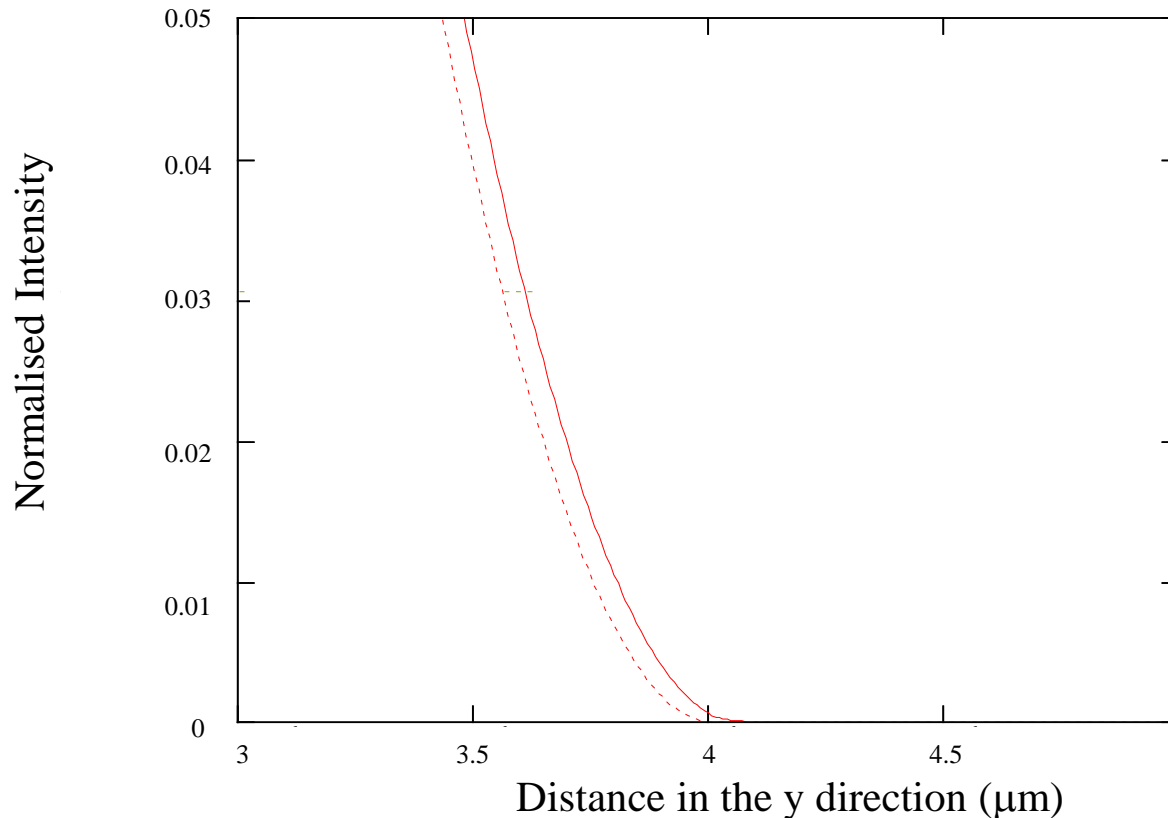
Distance in the y direction (μm)
TE and TM intensity profiles for $h = 1 \mu\text{m}$



Confinement is different for TE and TM modes

Polarisation dependence reduces with increasing waveguide height

Even for the larger waveguides, polarisation dependence is not removed



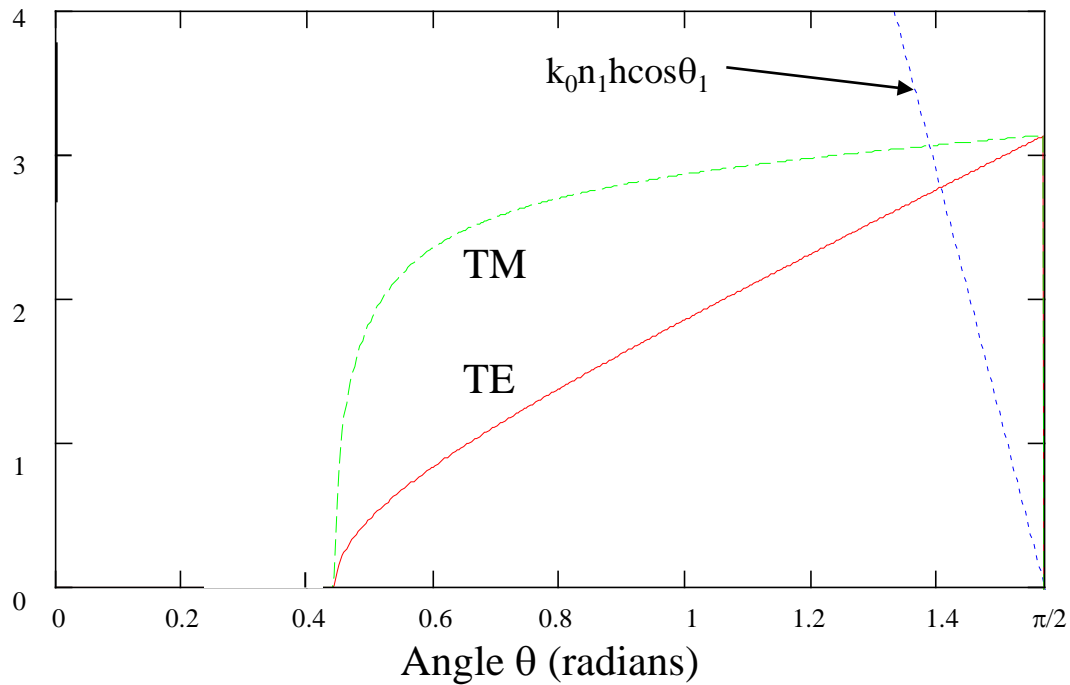
Enlarged view of the TE and TM mode profiles for $h=8\mu\text{m}$

The differences in confinement of the modes can be understood by considering again, the graphical solution of the eigenvalue equation. If we plot both the TE and the TM eigenvalue equations on the same axes, it is clear that the phase change on reflection for the TM mode is always greater than that for the TE mode.

Recall that the y -directed propagation constant was given by:

$$k_y = n_1 k_0 \cos \theta_1$$

Since θ_1 is larger for the TE mode, then 'cos θ_1 ' is smaller for the TE mode, and hence the propagation constant k_y is also smaller



**Solution of the TE and TM eigenvalue equation for
 $h = 1 \mu\text{m}$**

Surface scattering loss for different waveguide thickness and polarisation

Since TE and TM modes experience different degrees of optical confinement, the power at the waveguide core/cladding interfaces will be different. Furthermore the propagation angles are different, and one mode will experience more reflections at the core/cladding interface than the other. Consequently the loss due to scattering at the interfaces will also be different.

Previously we introduced the model of Tien to evaluate loss due to surface scattering:

$$\alpha_s = \frac{\cos^3 \theta}{2 \sin \theta} \left(\frac{4\pi n_1 (\sigma_u^2 + \sigma_\ell^2)^{\frac{1}{2}}}{\lambda_0} \right)^2 \left(\frac{1}{h + \frac{1}{k_{yu}} + \frac{1}{k_{yl}}} \right)$$

In order to utilise this model to compare TE and TM mode loss, we must first evaluate the decay constants k_{yu} for each mode (fundamental mode):

Waveguide thickness (μm)	TE ₀ $k_{yu} = \sqrt{\beta^2 - k_0^2 n_2^2}$	TM ₀ $k_{yu} = \sqrt{\beta^2 - k_0^2 n_2^2}$
0.3	13.481	11.968
1	15.0297	14.97322
5	15.2717	15.2712
8	15.2791	15.27896

Inserting these values into Tien's model produces the following predicted interface losses for a 1nm rms roughness at upper and lower interface:

Waveguide thickness (μm)	Interface scattering loss TE ₀ (dB/cm)	Interface scattering loss TM ₀ (dB/cm)
0.3	9.453	22.832
1	0.1966	0.25656
5	4.6×10^{-4}	4.9×10^{-4}
8	7.29×10^{-5}	7.58×10^{-4}

Despite the TM mode being more tightly confined, the model predicts a higher loss. This is suspicious, and highlights the importance of taking care when using attractively simple analytical models. Other authors have developed alternative models based upon roughness correlation functions, as well as optical power at the interface.

One such convenient model for the evaluating polarisation dependent loss coefficient, α_s due to interface scattering is:

$$\alpha_s = \Phi^2 \left(\frac{h}{2} \right) \cdot (n_{core}^2 - n_{cladd}^2)^2 \cdot \left(\frac{k_0^2}{8n_{core}} \right) \cdot \left(\frac{1}{N - n_{cladd}} \right) \cdot \sigma^2$$

where n_{core} is the refractive index of the core, n_{cladd} is the refractive index of the cladding, N is the effective index of the propagating mode, h is the waveguide thickness and $\Phi^2 \left(\frac{h}{2} \right)$ is the value of $[E_x(y)]^2$ in the TE case, or $\left(\frac{1}{n^2} \right) [H_x(y)]^2$ in the TM case, at the core/cladding interface.

If we again let the rms interface roughness be 1nm, we can compare interface scattering for different waveguide heights:

Waveguide thickness (μm)	Interface scattering loss TE_0 (dB/cm)	Interface scattering loss TM_0 (dB/cm)
0.3	2.15	3.8
1	0.108	0.063
5	1.13×10^{-3}	5×10^{-4}
8	2.85×10^{-4}	1.23×10^{-4}

Similarly we can compare scattering with interface roughness for a fixed waveguide thickness (e.g. $1\mu\text{m}$, fundamental mode):

Rms interface roughness (nm)	Interface scattering loss TE_0 (dB/cm)	Interface scattering loss TM_0 (dB/cm)
0.1	1.1×10^{-3}	6.3×10^{-4}
0.5	0.027	0.016
1	0.108	0.063
10	10.81	6.30
50	270	157

For comparison, consider interface scattering for a 5 μ m waveguide:

Rms interface roughness (nm)	Interface scattering loss TE ₀ (dB/cm)	Interface scattering loss TM ₀ (dB/cm)
0.1	1.13×10^{-5}	5×10^{-6}
0.5	2.83×10^{-4}	1.25×10^{-4}
1	1.13×10^{-3}	5×10^{-4}
10	0.113	0.05
50	2.83	1.25

It is clear that the polarisation dependant loss can become important if either the interface quality is not kept under control, or if the device is very long. For example the interface roughness of the silicon/buried oxide layer of commercially available SIMOX wafers is typically in the range 0.8nm to 3nm. Even if we assume the surface of such wafers is perfectly smooth, this model suggests a loss ranging from 0.04 dB/cm to 0.57 dB/cm for the fundamental TM mode of a 1 μ m waveguide.

Birefringence

The term ‘waveguide birefringence’ has emerged from the fibre and integrated optics fields to describe the difference between either propagation constants or effective indices of the two polarisation modes.

Since $\phi = \beta \cdot L$, then a change in propagation constant results in differential phase shift for TE and TM modes

Consider the phase changes in a $1\mu\text{m}$ planar waveguide, over a distance of 1mm:

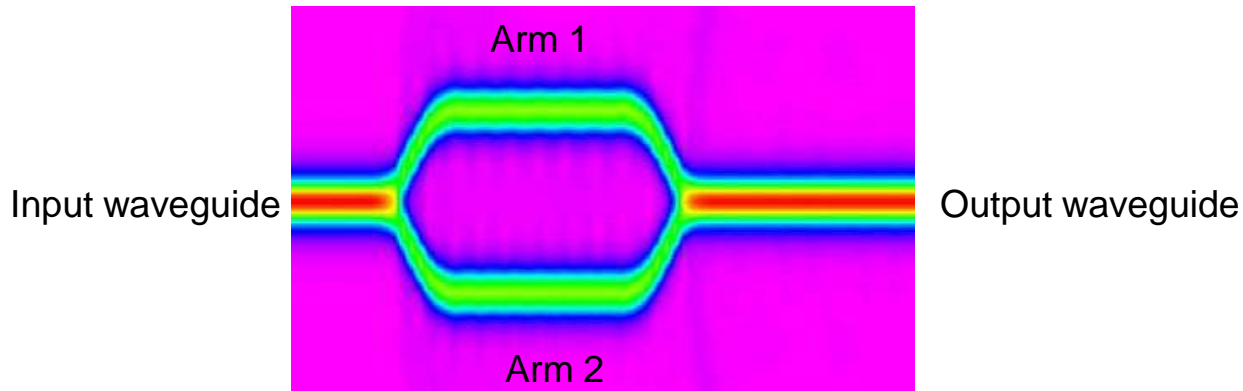
Mode number m	Propagation constant (TE)	Propagation constant (TM)	Phase change βL	
			TE	TM
0	$16.687\mu\text{m}^{-1}$	$16.636\mu\text{m}^{-1}$	5311.6π	5295.4π
1	$15.983\mu\text{m}^{-1}$	$15.769\mu\text{m}^{-1}$	5087.5π	5019.4π
2	$14.751\mu\text{m}^{-1}$	$14.227\mu\text{m}^{-1}$	4695.4π	4528.6π
3	$12.880\mu\text{m}^{-1}$	$11.822\mu\text{m}^{-1}$	4099.8π	3763.4π
4	$10.150\mu\text{m}^{-1}$	$8.506\mu\text{m}^{-1}$	3230.8π	2707.5π

Similarly the waveguide birefringence of rib waveguides can be studied by comparing propagation constants. This is equivalent to studying the effective index, N , because:

$$\beta = k_0 N_{wg}$$

where N_{wg} is the effective index of the propagating rib mode.

Let us consider the effect of waveguide birefringence on a Mach-Zehnder interferometer:

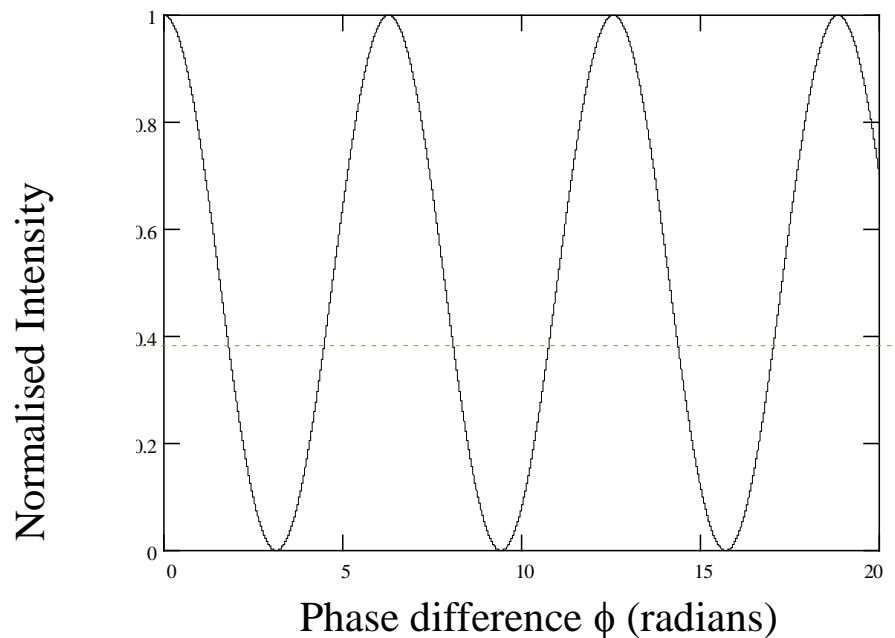


The intensity, I , in the output waveguide is given by the well known interferometer transfer function:

$$I = I_0 [1 + \cos(\beta_2 L_2 - \beta_1 L_1)]$$

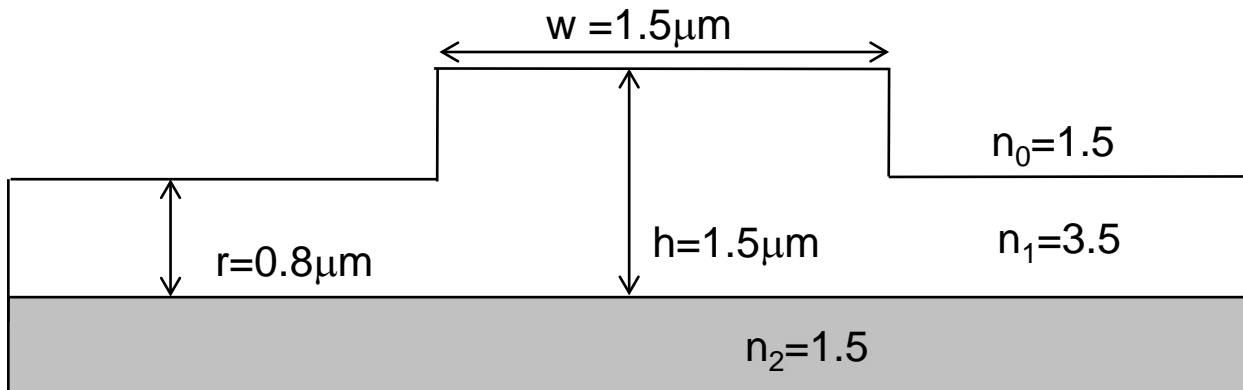
The term $(\beta_2 L_2 - \beta_1 L_1)$, represents the phase difference between the two arms of the interferometer.

Clearly if the two arms of the interferometer have identical propagation constants, the transfer function will be a maximum when the path length difference, $|L_2 - L_1|$, results in a phase difference of a multiple of 2π radians. Similarly the transfer function will have minimum when the phase difference is a multiple of π radians:



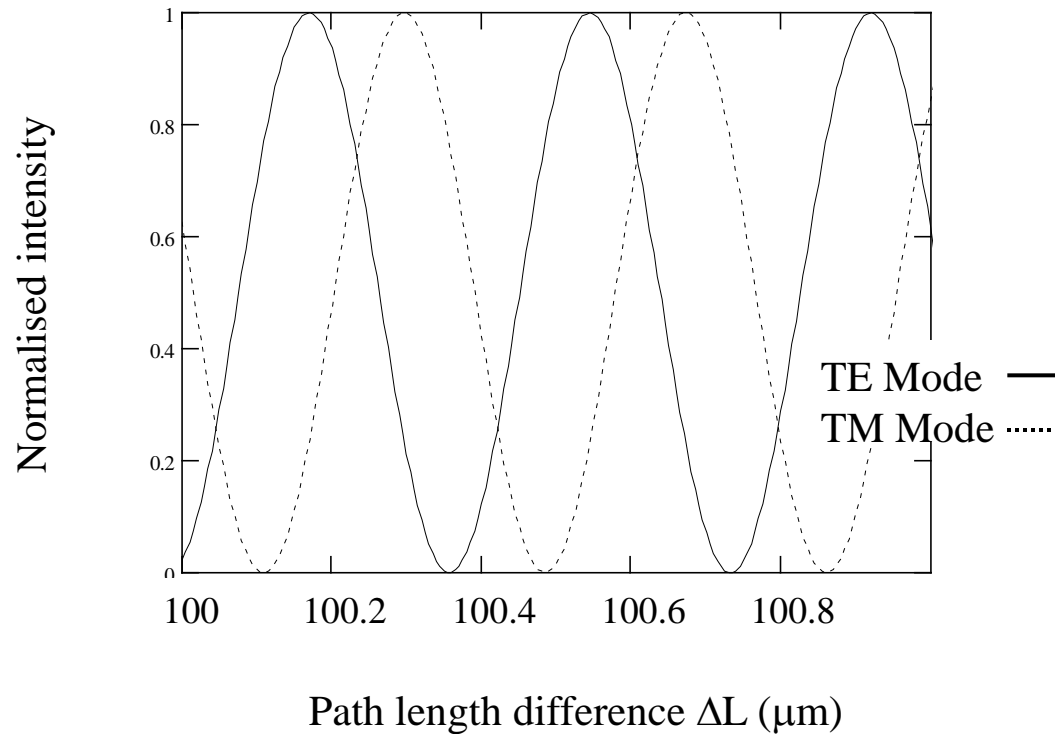
The normalised transfer function of a Mach Zehnder interferometer

To evaluate the polarisation dependence of the interferometer, we must use a specific example waveguide geometry:



The values of the effective indices are $N_{\text{wg}}(\text{TE}) = 3.4651$, and $N_{\text{wg}}(\text{TM}) = 3.4607$. This corresponds to propagation constants of $\beta(\text{TE}) = 16.748 \mu\text{m}^{-1}$ and $\beta(\text{TM}) = 16.726 \mu\text{m}^{-1}$.

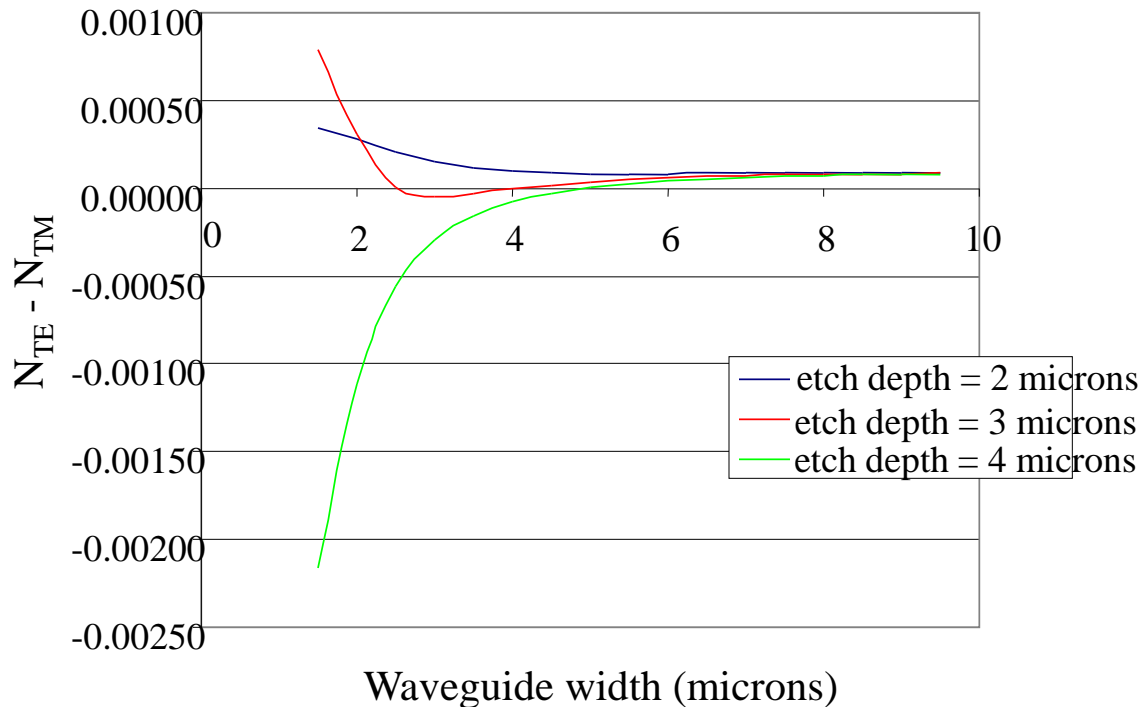
The phase difference between TE and TM modes becomes significant with increasing path length difference between the interferometer arms. At about $100\mu\text{m}$ the situation is serious:



Interferometer transfer function for TE and TM modes

Can we make polarisation independent rib waveguides?

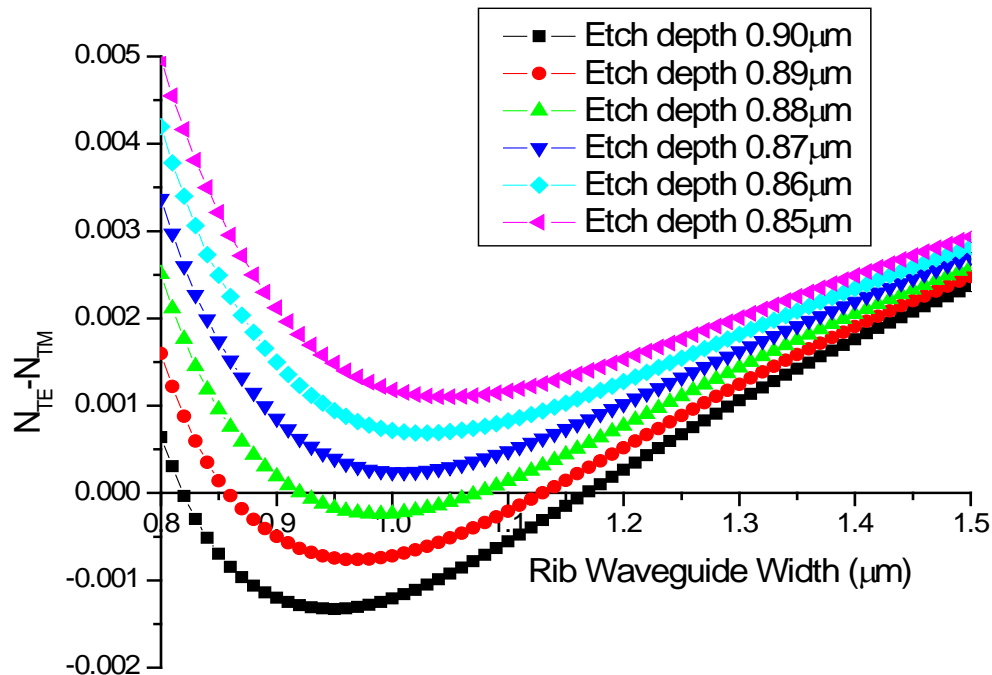
Consider the variation in birefringence with rib etch depth, for a 5 μm guiding layer thickness:



Birefringence of a 5 μm waveguide (height) for various etch depths and waveguide widths

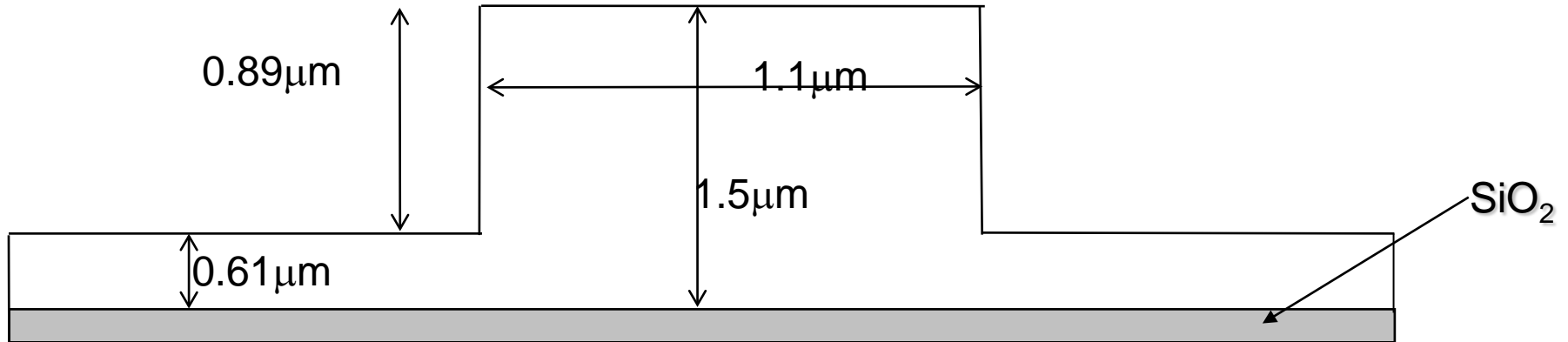
Since the etch depth of 3 microns cuts the zero birefringence axis it is possible to choose an etch depth such that the waveguide modes both have the same effective index.

However, consider a smaller rib waveguide, with a rib height of $1.5\mu\text{m}$:



Polarisation dependence of a $1.5\mu\text{m}$ high rib waveguide

The required etch depth to cut the zero birefringence axis is $0.88\mu\text{m}$. This results in the following waveguide design:



Rib Waveguide Geometry

Such an 'over-etched' rib waveguide may violate the single-mode condition

Hence great care is required in the design of the waveguide, particularly at small dimensions

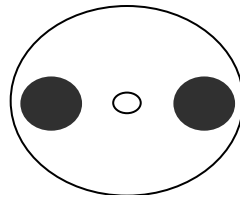
- Waveguide materials
- Planar waveguides
- Rib and ridge waveguides
- Strip waveguides
- Losses
- Polarisation issues
- Effect of stress

The effect of stress

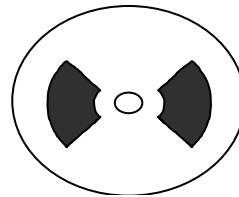
The semiconductor processing laboratory, together with device packaging, offer numerous opportunities of introducing stress to semiconductor devices and wafers. For optical waveguides this means that the potential exists for stress induced changes in refractive index of the waveguiding layer.

This is particularly problematic when the resulting strain field is directional, because this will result in directional changes in the optical properties of the waveguide. This commonly translates to polarisation dependent changes in propagation characteristics and/or losses.

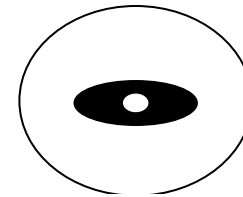
Consider an application where stress in an optical system is made a virtue. Such a case is the polarisation maintaining fibre. A directional stress field is deliberately introduced to ensure that orthogonal polarisation modes propagate with significantly different propagation constants. That is to say the fibre is birefringent.



(a) Panda Fibre



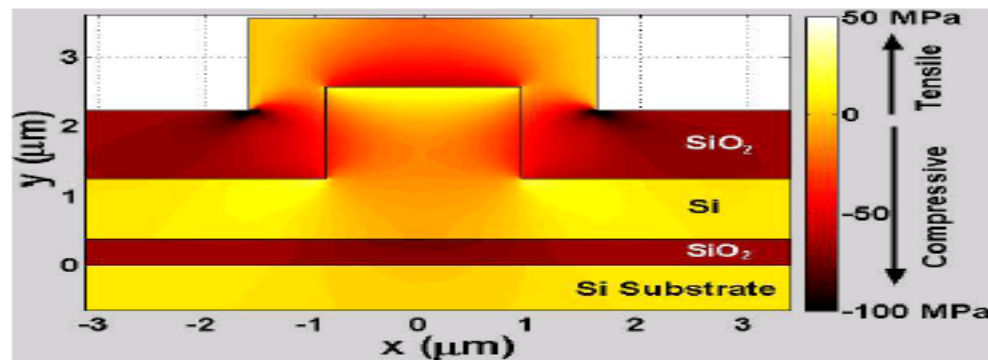
(b) Bow-tie fibre



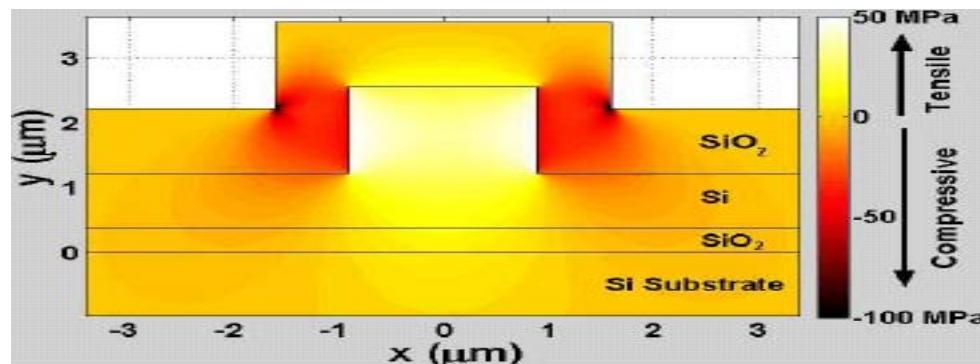
(c) Elliptical jacket
fibre

Three types of high birefringence fibre formed by stress inducing inclusions

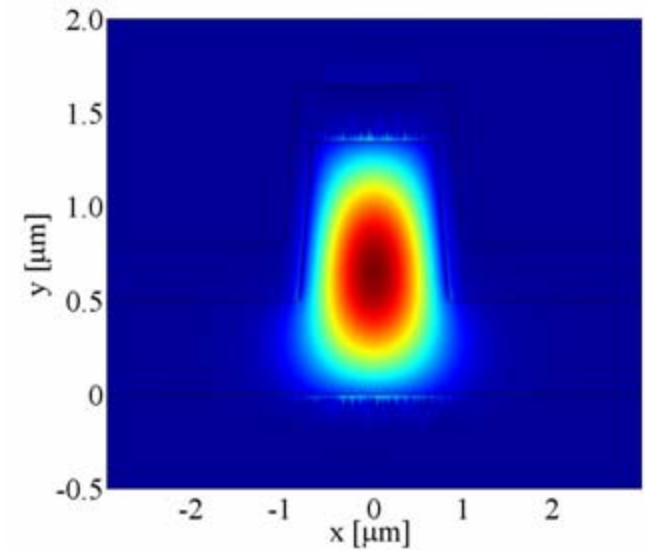
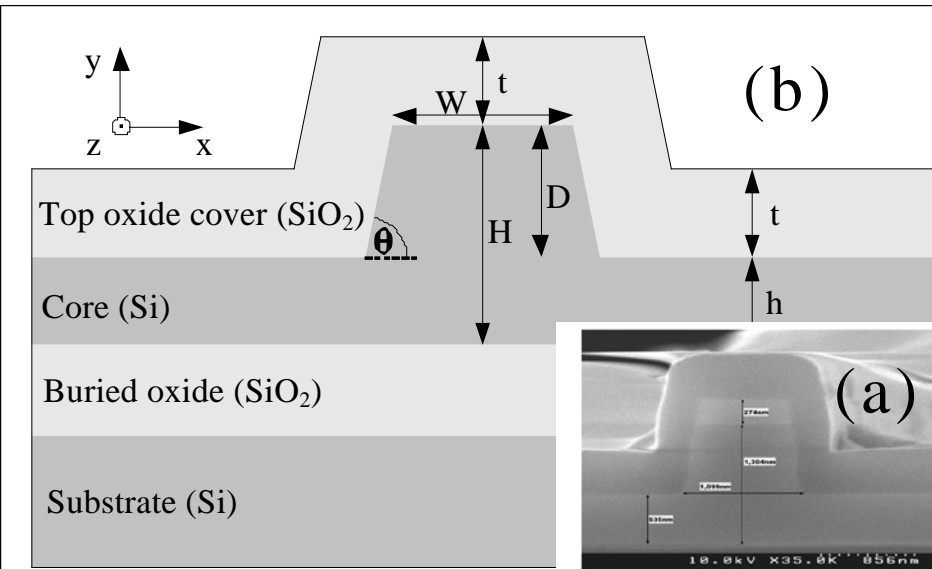
Silica (SiO_2) cladding can be used advantageously to allow additional control of the birefringence in SOI waveguides. For a rib height of $2.2\mu\text{m}$, a rib width of $1.83\mu\text{m}$, and a silica cladding thickness of $1\mu\text{m}$, the following diagrams show stress in the x and y directions for an upper cladding under compressive strain of -100MPa :



Strain in the x-direction



Strain in the y-direction



Field profile in an SOI rib waveguide

- Rib Waveguide cross-section view
a) SEM structure; b) modeled structure
- Operating wavelength: $\lambda = 1.55\mu\text{m}$
- Waveguide height: $H = 1.35\mu\text{m}$

- Modal birefringence

$$\Delta N_{eff} = N_{eff}^{TE} - N_{eff}^{TM}$$

- Stress relations

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} + \begin{bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ \alpha\Delta T \end{bmatrix}$$

- Stress-Induced birefringence

$$\Delta N_x = N_x - N_0 = -C_1\sigma_x - C_2(\sigma_y + \sigma_z)$$

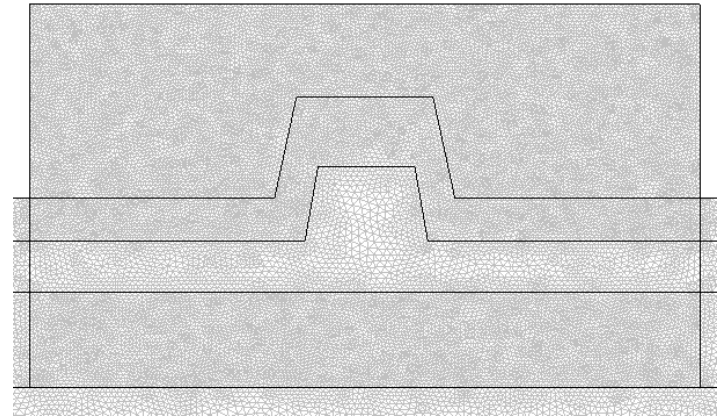
$$\Delta N_y = N_y - N_0 = -C_1\sigma_y - C_2(\sigma_x + \sigma_z).$$

- Maxwell's equation

$$\nabla \times \varepsilon_r^{-1} \times \vec{H} = k_0^2 \vec{H}, \vec{H} = \vec{H}(x, y)e^{j(\omega t - \beta z)}$$

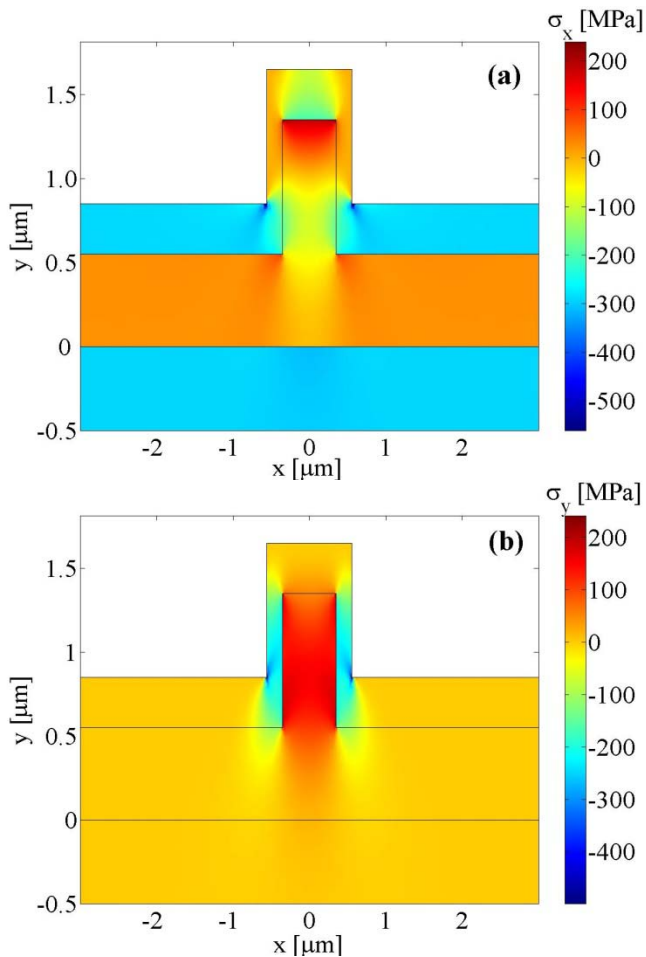
- Boundary conditions

$$\mathbf{n} \cdot (\mathbf{H}_1 - \mathbf{H}_2) = 0$$

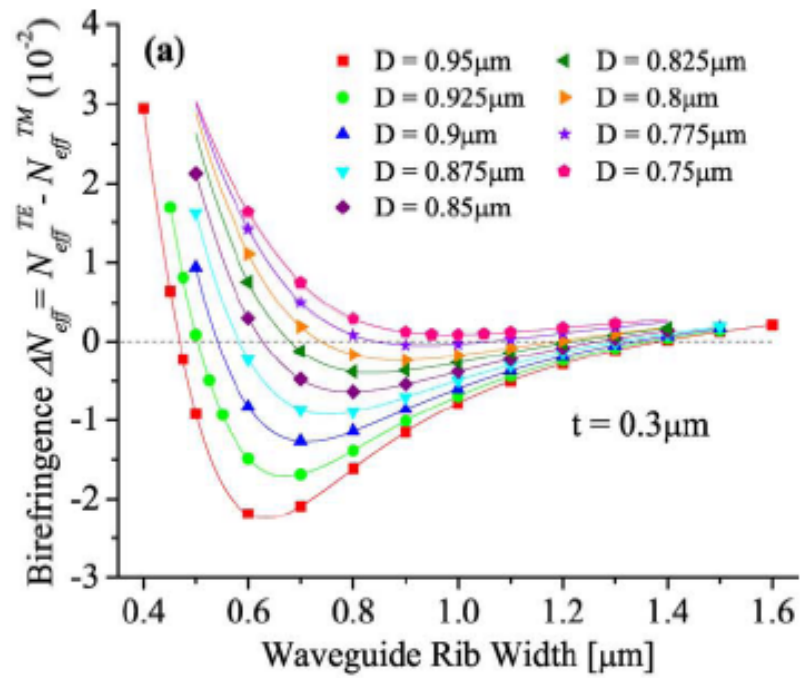


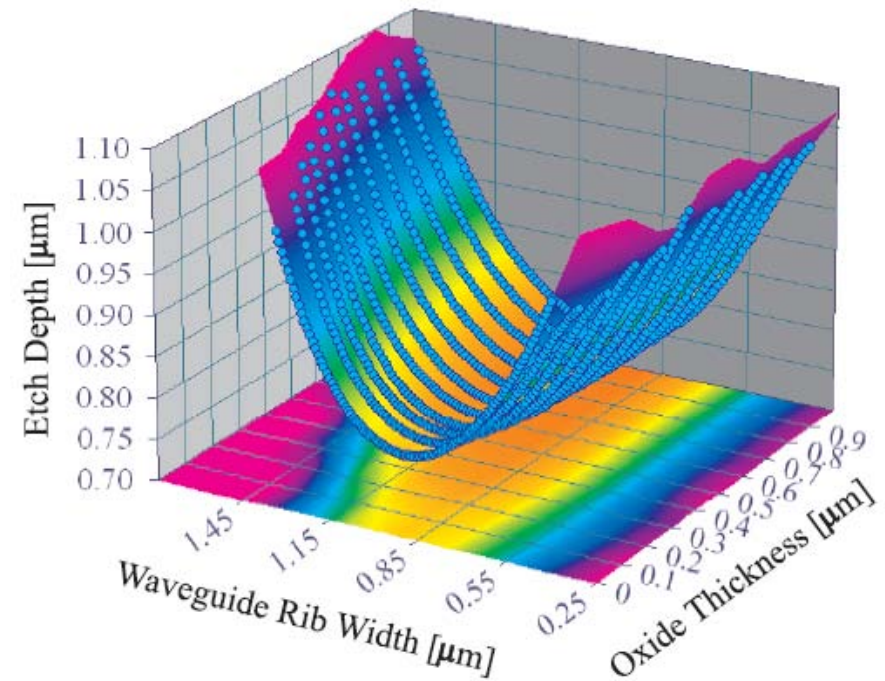
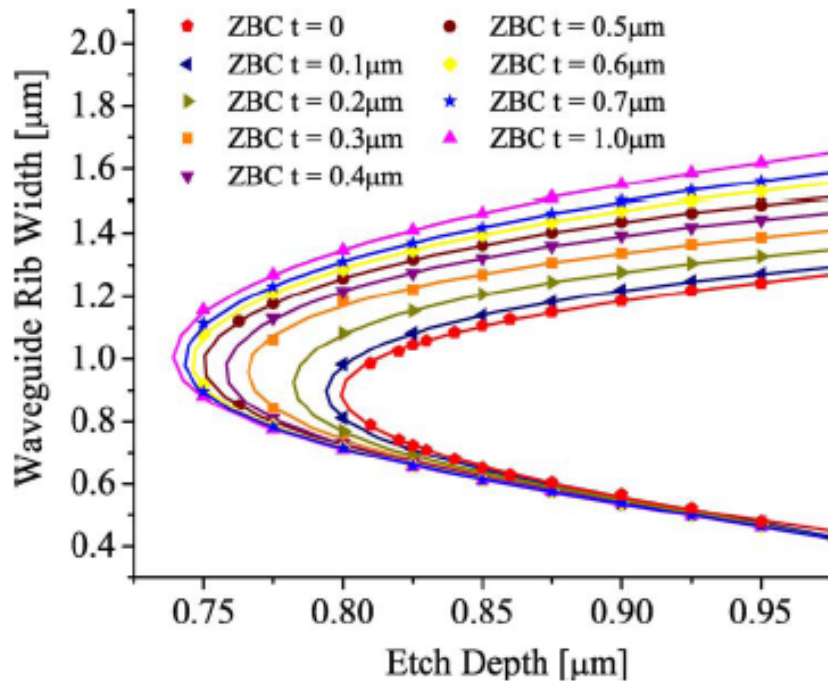
- Numerical methods
BPM & FEM

- Effective mode index
 $N_{eff} = \beta/k_0$



- Stress field distribution
 - a) in the x direction, σ_x
 - b) in the y direction, σ_y
- Modal waveguide birefringence





- Zero birefringent condition as a function of waveguide parameters
- Zero birefringent surface
- Vertical rib sidewalls $\theta = 90^\circ$

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